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INTEGRAL APPROACH TO ASSESSING ENERGY LOSSES DURING THE MOTION OF A TRACTION VEHICLE WITH A HYDRO-MECHANICAL TRANSMISSION

Purpose. To develop an integral approach for assessing energy losses arising during the motion of a traction vehicle equipped with a split-power hydro-mechanical transmission, with particular attention to power circulation in the planetary summation mechanism.

Methodology. A theoretical model of a split-power hydro-mechanical transmission was developed to achieve this objective. The model incorporates planetary gear kinematics, power distribution between transmission branches, and internal power circulation effects. An integral indicator is introduced to evaluate energy losses over the operating speed range of the traction vehicle.

Findings. The results show that power circulation and the associated energy losses strongly depend on transmission control parameters and operating speed. In certain regimes, circulating power reaches significant levels, leading to a noticeable reduction in overall transmission efficiency. The proposed approach makes it possible to identify parameter regions in which integral energy losses are minimized.

Originality. The originality of the study lies in the formalization of power circulation as an independent energy component of the traction vehicle motion process and in the development of an integral loss assessment approach. This allows the transition from point-based efficiency evaluation to a comprehensive analysis of transmission energy behaviour across the entire operating range.

Practical value. The obtained results can be used in the design and tuning of hydro-mechanical transmissions for traction vehicles to reduce energy losses, decrease thermal loading of planetary mechanisms, and increase transmission durability. The proposed integral approach to loss assessment provides a practical tool for selecting hydraulic machine parameters and operating regimes under real operating conditions.

Keywords: *split-power transmission, power circulation, energy losses, planetary mechanism, vehicle*

Introduction. Hydro-mechanical transmissions (HMT) of the split-power type are widely used in traction mobile machines, including mine diesel locomotives, mining dump trucks, agricultural tractors and transport-traction modules [1]. Their key advantage is the ability to continuously change the gear ratio with high overall efficiency, achieved by combining a mechanical branch with high efficiency and a hydrostatic branch that provides speed control [2].

At the same time, one of the most significant problems of split-power HMT is the circulation of power in the planetary summing mechanism (gearbox), which arises from the interaction of flows from the hydraulic and mechanical branches. Circulating power leads to increased heat generation, overloading of gears, additional load on supports and bearings, and a decrease in energy efficiency and transmission life. It is known that in some operating modes the value of circulating power can reach values comparable to half the input power of the motor [3], which makes its control critical for heavy machines operating in conditions of limited heat removal (for example, in underground mines).

Despite the presence of a significant number of papers devoted to the kinematics and force analysis of HMT, there is practically no formalized criterion that allows quantitative assessment of circulation losses in the operating range of speeds and parameters of the hydraulic system. Available models mainly consider the instantaneous efficiency of the branches or characterize the operation of the transmission at individual points, without providing the possibility of an integral assessment of the energy behaviour of the system.

Literature review. Double-flow hydrostatic-mechanical transmissions belong to the split-power HMT (Hydro-Mechanical Transmission) class, the design basis of which is the summation of the power of two branches – mechanical and hydraulic – in a planetary mechanism. A significant contribution to the development of the theory and design of such transmissions was made in works devoted to transmissions for tractors, road construction and traction equipment: the developments of Fendt (Vario), Claas (CMATIC), John Deere (AutoPowr), Sauer-Danfoss (HMT), etc. are known. In these systems, the main effect is achieved due to the high mechanical share of power, which allows to significantly increase efficiency compared to fully hydrostatic transmissions.

The literature has formed a fundamental basis for understanding power distribution and efficiency in hydro-mechanical transmissions. Studies by Zhu and Gao [4], Bietresato, et al. [5] and Yang, et al. [6] showed the relationship between transmission structure, engine performance and HMT/HMCVT energy modes. Zhu, et al. [7] and Zhang, et al. [8] discuss in detail the mode change and the influence of physical parameters of the transmission on the efficiency of power transfer.

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Particular attention of researchers is drawn to the circulation component of power ("power reflux", "power cycle", "parasitic power"), peculiar to split-power HMT. One of the first works to systematically describe this effect is the paper by He, et al. [9], where it is shown that in some HMCVT architectures the circulating ("cycling") power can reach 30–40 % of the input power. Similar conclusions are made in the studies by Xia and Sun [10], which introduce the concept of power-cycling CVT and provide a mathematical apparatus for assessing circulation modes.

More recent studies consider both the static and dynamic behaviour of power recirculating systems. Wang, et al. [11] and Wang & Ge [12] studied the nonlinear dynamics of power-reflux systems, including possible resonances, bifurcations and the influence of transients. These papers emphasize that power circulation is not only an energy factor, but also a dynamic factor that influences vibration, noise, element load and thermal operating conditions of the planetary gear set.

Significant contributions to the understanding of optimal power allocation were made by Xia, et al. [13], Liu, et al. [14], Cheng, et al. [15]. The works of these authors discuss the issues of optimizing HMCVT parameters, constructing efficiency maps and choosing the working volumes of hydraulic machines to minimize heat and hydraulic losses. These models are prototypes for the development of generalized energy criteria [16] applicable to the comparison of different transmission configurations.

The seminal work by Zhang, Sun and Qin [17] considers power reflux HMT and first proposes a method for optimizing power transmission parameters based on power circulation estimation. However, the proposed methods cover only individual modes and do not provide an integral criterion that would allow estimating circulation losses over the entire speed range.

Modern research is aimed at further systematization and expansion of power-split transmission models. Works by Zhao, et al. [18], Wang, et al. [19] and Sun, et al. [20] show that assessing the effectiveness of HMT/HMCVT requires considering the combined influence of power distribution, hydrostatic losses and circulation flows, as well as transient processes that occur when changing the driving mode.

It should be noted that circulation and parasitic power losses in power-split hydromechanical transmissions are formed not only by the structure and parameters of the transmission itself, but also by the nature of external operating loads. Works [21, 22] show that the mechanical and dynamic properties of railway infrastructure elements, including the rigidity of the ballast layer, deformation characteristics of sleepers and track inhomogeneities, significantly affect the distribution of loads in the wheel-rail system. These loads determine the actual movement modes of diesel locomotives and form disturbing influences transmitted to the power plant and transmission. Additionally, in [23], the behaviour of railway infrastructure elements under quasi-realistic loads was studied, which confirms the need to consider energy modes and circulation losses in the transmission of diesel locomotives, considering external operating conditions that determine the level and nature of input loads.

In Ukraine, an independent scientific direction has been formed, focused on the tasks of calculation, design and analysis of hydraulic volumetric and hydraulic volumetric-mechanical systems of transport and technological machines [24]. A significant contribution to the development of this area was made by the scientific school of Professor V. B. Samorodov (National Technical University "Kharkiv Polytechnic Institute"), within the framework of which research was carried out on the calculation of the parameters and dynamics of volumetric hydraulic drives, the analysis of leaks and energy losses in hydraulic machines, the development of stepless hydro-mechanical transmissions for tractor equipment, as well as on the issues of algorithmizing and automation of the design and control of power plants of transport vehicles [25, 26].

The analysis of the literature shows that at certain combinations of speeds and parameters of hydraulic machines in the planetary gear, a circulating power flow arises, which is part of the energy that is not transmitted to the output, but "goes in a circle" between the branches, creating overheating and increasing the force load of the elements. This effect is qualitatively described in a number of analysed works; however, a strict quantitative circulation model is rarely proposed and, as a rule, is limited to the consideration of individual points or special modes (for example, near-zero velocity mode).

An additional complexity lies in the fact that the power circulation: is not a constant value, depends on the speed state of the planetary gear, is sensitive to the ratio of the working volumes of the pump and motor, varies depending on the share of mechanical and hydraulic paths in the transmitted power, can be extreme in certain speed zones, including during transient modes.

The existing approaches do not offer an integral criterion similar to those described in [16], which allows one to evaluate the energy intensity of the circulation over the entire speed range and compare different transmission settings with each other. A detailed analysis of modern publications shows the presence of the following methodological gaps: the lack of a formalized description of the circulating power in two-flow HMT as a separate energy parameter; the lack of an integral measure that would allow assessing the circulation of power not at points, but in the operating speed range; insufficient connection between the design parameters of the HMT (the ratio of the working volumes of hydraulic machines, the geometry of the planetary gear) and the energy consequences of circulation; lack of criteria for choosing a rational share of mechanical and hydraulic flows in order to reduce overheating and increase transmission life.

The purpose of this research is to develop a theoretical model of a split-power hydro-mechanical transmission with a summing planetary gear, to formalize circulating power flows and to construct an integral criterion of energy efficiency applicable for selecting the parameters of hydraulic machines and tuning the transmission of a mine diesel locomotive.

Methods. A split-power hydro-mechanical transmission (HMT) of a mine diesel locomotive is considered as a modelling object. The power transmission of a diesel locomotive consists of:

- mechanical branch: motor – gear reducer – sun gear of a simple planetary gear set;

- hydraulic branch: motor – adjustable axial piston pump, hydraulic motor – gearbox – planetary ring gear;

- transmission output – planetary gear carrier, connected to the driving wheels of the diesel motor.

To form a basic calculation model and identify the influence of power circulation on the energy performance of the transmission, the work considers the simplest class of modes. It is assumed that the movement of the traction machine is carried out in stationary steady-state modes, without considering the processes of acceleration and transient operating modes of the motor and power transmission.

Movement is considered along a horizontal section of the path; external resistance to the movement of the train is not included in the model, since the purpose of the study is limited to the analysis of energy processes occurring inside the transmission. Within the framework of the adopted approach, it is not the traction dynamics of the vehicle that is considered, but the energy state of the transmission in steady states under given kinematic conditions. It is assumed that the external load ensures the establishment of a stationary mode at a given angular velocity of the output link, and the distribution and interaction of power flows inside the transmission are determined by its structure and control parameters. This approach makes it possible to analyse the circulating energy processes in the planetary mechanism regardless of the specific profile of the resistance to motion and focus on the internal energy interactions in a power-split HMT. The efficiency coefficients of the mechanical branch η_g, the hydraulic branch η_h and the planetary gear η_p within the framework of the model under consideration are assumed to be constant, which corresponds to the analysis of steady-state conditions and allows us to focus on the influence of power circulation on the energy performance of the transmission.

Considering the complexity of the structure of a power-split HMT and the significant number of parameters and notations used, the following is a set of basic symbols and units of measurement used in the mathematical model. This makes it possible to simplify the perception of subsequent relationships and avoid ambiguities when presenting the results. The following notations are used: ω_d is angular velocity of the diesel motor, rad/s; ω_s is angular velocity of the sun gear, rad/s; ω_r is angular velocity of the ring gear, rad/s; ω_c is angular velocity of the carrier (transmission output), rad/s; $M_d(\omega_d)$ is motor torque, Nm; $N_d(\omega_d)$ is motor power, W; Z_s, Z_r is number of teeth of sun and crown gears, dimensionless; $k_z = \frac{Z_s}{Z_r}$ is planetary gear parameter, dimensionless; i_s is gear ratio of the mechanical branch motor – sun gear, dimensionless; i_h is gear ratio of the motor – ring gear branch, dimensionless; q_p, q_m are working volumes of the pump and motor, m^3/rad; $\beta(\lambda)$ is fraction of motor power directed to the mechanical branch, dimensionless, $0 \le \beta \le 1$; N_s, N_r, N_c are powers supplied to the sun, corona and carrier, W; N_{circ} is circulation power in the planetary gear, W; $\eta_{tr}(\lambda, \omega_c)$ is overall transmission efficiency, dimensionless; $J(\lambda)$ is integral criterion of circulation losses, dimensionless (after normalization).

When forming a kinematic model of a power-split hydro-mechanical transmission, the planetary gear set is considered as a simple planetary mechanism of the "sun gear – satellites – ring gear" type. For the specified configuration, the following kinematic relationship is satisfied between the angular velocities of its links

$$\omega_c = (1 - k_z)\omega_s + k_z\omega_r, \quad (1)$$

where ω_s stands for sun gear speed; ω_r stands for ring gear speed; ω_c stands for carrier speed; k_z stands for parameter determined by the ratio of the number of teeth according to the formula

$$k_z = \frac{Z_s}{Z_r}.$$

To establish the kinematic relationship between the motor speed and the angular velocities of the planetary gear units, we consider separately the mechanical and hydraulic branches of the transmission.

In the mechanical branch, rotation from the motor is transmitted to the sun gear through a gearbox with a constant gear ratio i_s, resulting in the following relation

$$\omega_s = i_s\omega_d, \quad (2)$$

where i_s is the constant gear ratio of the gearbox from the motor to the sun gear (in numerical calculations $i_s = 0.05$, is taken, but in the analytical model we leave it as a parameter).

In the hydraulic branch, the rotation speed of the ring gear is determined through the parameters of the hydrostatic transmission. The flow through the pump and motor is described by the expressions

$$Q_p = q_p\omega_d; \quad Q_m = q_m\omega_m,$$

where ω_m stands for angular velocity of the hydraulic motor shaft.

For a closed hydrostatic circuit, neglecting leaks, the pump and motor costs are equal, which means that

$$Q_p = Q_m \Rightarrow q_p\omega_d = q_m\omega_m. \quad (3)$$

Introducing a dimensionless control parameter

$$\lambda = \frac{q_p}{q_m},$$

we get the relation

$$\omega_m = \lambda\omega_d. \quad (4)$$

Next, rotation from the hydraulic motor is transmitted to the ring gear through a gearbox with gear ratio i_h, which leads to the relation

$$\omega_r = i_h\omega_m = i_h\lambda\omega_d. \quad (5)$$

The relation for the angular velocities of the sun and ring gears allows us to proceed to determining the speed of the output link of the transmission – the planetary gear carrier. Substituting expressions (2 and 5) into the kinematic relation of the planetary gear set (1), we obtain

$$\omega_c = (1 - k_z)i_s\omega_d + k_z i_h\lambda\omega_d = \omega_d[(1 - k_z)i_s + k_z i_h\lambda].$$

Thus, the speed of the carrier, and therefore the speed of movement of the traction machine, is determined by the transmission parameters and the control mode of the hydrostatic branch. For a compact repre-

sentation of the obtained kinematic dependence and convenience of further analysis, we introduce the kinematic transmission factor $\Phi(\lambda)$, which considers the influence of the control parameter and the structure of the power transmission

$$\Phi(\lambda) = (1 - k_z)i_s + k_z i_h \lambda. \quad (6)$$

Considering the introduced multiplier, the relationship between the angular velocity of the motor and the speed of the transmission output relation can be written in the form

$$\omega_c = \Phi(\lambda)\omega_d. \quad (7)$$

The resulting expression specifies a direct kinematic relationship between the motor operating mode and the speed of the traction machine at a fixed value of the control parameter λ.

From (7) the inverse relationship directly follows

$$\omega_d = \frac{\omega_c}{\Phi(\lambda)}. \quad (8)$$

In the following, expression (8) is used to bring the motor characteristics to the parameters of vehicle motion and transmission.

To describe the dependence of torque on angular velocity, a generalized approximation formula of the Watson-Hindmarsh type is used

$$M_d(\omega_d) = M_{\max}\left(\frac{\omega_d}{\omega_M}\right)^a \left[b - \left(\frac{\omega_d}{\omega_M}\right)^c\right], \quad (9)$$

where $M_{\max}$ stands for maximum torque, Nm; ω_M stands for angular velocity at which maximum torque is achieved, rad/s; a, b, c stand for dimensionless characteristics shape parameters.

For a diesel motor with a power of about 100 kW, the typical parameter values used in the numerical examples are taken in the following ranges: $M_{\max}$ = 500–600 Nm; $\omega_M = 2\pi \cdot$ (1,300–1,500) rpm = 136–157 rad/s; a = 1, b = 1.2–1.4, s = 1.8–2.5 .

When going beyond the effective speed range, the expression in square brackets in (9) can take on negative values. In this case, the motor torque is assumed to be zero, which reflects the physical impossibility of stable operation of the motor in this mode

$$M_d(\omega_d) = \max\left\{0, M_{\max}\left(\frac{\omega_d}{\omega_M}\right)^a \left[b - \left(\frac{\omega_d}{\omega_M}\right)^c\right]\right\}. \quad (10)$$

The current motor power is determined by the standard relationship, Wt

$$N_d(\omega_d) = M_d(\omega_d)\omega_d, \text{ [N_d]}. \quad (11)$$

Considering the kinematic relationship between the motor speed and the speed of the transmission output link (8), the motor power can be presented as a function of the vehicle speed and the control parameter

$$N_d(\omega_c, \lambda) = M_d\left(\frac{\omega_c}{\Phi(\lambda)}\right)\frac{\omega_c}{\Phi(\lambda)}. \quad (12)$$

It should be noted that in expression (12) the motor torque and angular velocity refer to the same physical argument – the actual angular velocity of rotation of the motor.

Since the analysis is carried out in the coordinates of the transmission output link, the angular velocity of the motor ω_d is expressed through the carrier speed ω_c using the kinematic factor $\Phi(\lambda)$. As a result, the motor characteristic $M_d(\omega_d)$ is reduced to the vehicle speed by substituting $\omega_d = \omega_c/\Phi(\lambda)$.

Thus, expression (12) is a standard relationship for motor power $N_d = M_d\omega_d$, written in a form convenient for further analysis of energy processes in the transmission.

In a split-power hydrostatic-mechanical transmission, the power developed by the motor is transmitted to the planetary gearset along two parallel paths – mechanical and hydraulic. Depending on the transmission control mode, the proportion of power sent to each path may vary.

To describe the power distribution, we introduce the dimensionless function $\beta(\lambda)$, which determines the share of motor power transmitted through the mechanical branch. Then the input power supplied to the planetary gear sun gear can be written as

$$N_{s,in} = \beta(\lambda)N_d. \quad (13)$$

Accordingly, the remaining part of the motor power is directed to the hydraulic branch and supplied to the pump

$$N_{p,in} = [1 - \beta(\lambda)]N_d. \quad (14)$$

The function $\beta(\lambda)$ reflects the nature of the power distribution between the transmission branches and in the general case is determined by the control parameter λ. Within the framework of this model, the specific form of the function $\beta(\lambda)$ is not fixed; the very fact of its dependence on the regulatory mode is fundamental. In the simplest version, $\beta(\lambda)$ can be specified by a linear or piecewise linear function.

When transmitting power along each of the branches, part of the energy is lost due to internal losses, which, within the framework of the adopted model, are considered using the efficiency of the mechanical and hydraulic branches.

The power supplied to the sun gear after passing through the mechanical branch is given by

$$N_s(\lambda, \omega_c) = \eta_g\beta(\lambda)N_d(\omega_c, \lambda). \quad (15)$$

Similarly, the power supplied to the ring gear after passing through the hydraulic branch is equal to

$$N_r(\lambda, \omega_c) = \eta_h[1 - \beta(\lambda)]N_d(\omega_c, \lambda). \quad (16)$$

Thus, two powers are simultaneously supplied to the planetary gear – from the mechanical and hydraulic branches. Their sum determines the energy state of the input of the planetary mechanism.

The total power supplied to the planetary gear can be presented as

$$N_s + N_r = [\eta_g\beta(\lambda) + \eta_h[1 - \beta(\lambda)]]N_d. \quad (17)$$

To simplify further calculations and clearly separate the influence of power distribution and losses in branches, we introduce an effective power transfer coefficient

$$\Xi(\lambda) = \eta_g\beta(\lambda) + \eta_h[1 - \beta(\lambda)], \quad (18)$$

where $\Xi(\lambda)$ stands for the effective power transfer coefficient of the branches to the planetary gear, which char-

acterizes the combined influence of the mechanical and hydraulic branches on the level of power supplied to the planetary gear, without considering losses in the planetary mechanism itself.

Then the total power at the planetary gear input can be written in compact form

$$N_s + N_r = \Xi(\lambda) N_d(\omega_c, \lambda). \quad (19)$$

When summing the power flows in a planetary gear, not all the power input is converted into useful power output. In certain modes, part of the energy is closed inside the planetary gear, forming the so-called circulating power, which is not transmitted to the carrier, but leads to additional losses and heating of transmission elements.

To quantitatively describe this phenomenon, we introduce the dimensionless power circulation coefficient $b(\lambda, \omega_c)$, $0 \le b \le 1$, which shows what proportion of the total power supplied to the planetary gear goes into internal circulation. It should be noted that in the framework of this work, the circulation component of power is considered as an integral characteristic of the energy state of the planetary mechanism, reflecting the cumulative effect of the interaction of the mechanical and hydraulic branches at given operating parameters. The specific functional dependence of the circulation coefficient on the control parameter and speed state in this model is of an approximation nature and is used for illustration and comparative analysis of energy trends. A detailed derivation of circulating power from the balance of torques and pressures, as well as its identification based on the parameters of a specific transmission, are beyond the scope of this study.

The coefficient $b(\lambda, \omega_c)$ depends both on the transmission control mode and on the speed state of the planetary mechanism, which reflects the nonlinear nature of the circulation processes.

Then the circulating power can be determined by the expression

$$\begin{aligned} N_{circ}(\lambda, \omega_c) &= b(\lambda, \omega_c)(N_s + N_r) = \\ &= b(\lambda, \omega_c)\Xi(\lambda) N_d(\omega_c, \lambda). \end{aligned} \quad (20)$$

Of the total power supplied to the planetary gear, the part corresponding to the circulation component does not participate in the formation of useful output power. The remaining share of power is transferred to the carrier and is further reduced due to losses in the gearing of the planetary mechanism.

Considering the mechanical efficiency of the planetary gear set η_p, the net power at the transmission output is determined by the expression

$$\begin{aligned} N_c(\lambda, \omega_c) &= \eta_p[1 - b(\lambda, \omega_c)](N_s + N_r) = \\ &= \eta_p[1 - b(\lambda, \omega_c)]\Xi(\lambda) N_d(\omega_c, \lambda). \end{aligned} \quad (21)$$

Overall transmission efficiency is defined as the ratio of net output power to motor power

$$\eta_{\text{tr}}\left(\lambda, \omega_c\right) = \frac{N_c\left(\lambda, \omega_c\right)}{N_d\left(\omega_c, \lambda\right)}. \quad (22)$$

Substituting expression (21) into (22), we obtain the formula for the overall transmission efficiency

$$\eta_{tr}(\lambda, \omega_c) = \eta_p[1 - b(\lambda, \omega_c)]\Xi(\lambda). \quad (23)$$

Formula (23) clearly demonstrates the separate influence of various factors on the energy efficiency of the transmission: η_p characterizes mechanical losses in the planetary gearset; $b(\lambda, \omega_c)$ reflects the influence of circulating power that is not converted into useful power; $\Xi(\lambda)$ takes into account losses in the mechanical and hydraulic branches up to the planetary mechanism.

From (23) it follows that a decrease in the overall efficiency of the transmission occurs both with an increase in the circulating component of power and with an increase in the share of the hydraulic transmission path, which is characterized by lower efficiency values compared to the mechanical branch.

Instantaneous values of the circulating power $N_{circ}(\lambda, \omega_c)$) make it possible to analyse the energy state of the transmission in individual modes, but do not make it possible to compare different options for tuning the transmission over the entire operating speed range. To solve this problem, it is advisable to use an integral estimate that considers the behaviour of circulation processes over the entire speed range. Let us consider the operating range of angular speeds of the carrier

$$\omega_{c,\min} \le \omega_c \le \omega_{c,\max}.$$

To transition to a dimensionless form and eliminate the influence of the absolute level of motor power, we introduce normalized circulating power, defined as the ratio of circulating power to the current motor power

$$\psi\left(\lambda, \omega_c\right) = \frac{N_{circ}\left(\lambda, \omega_c\right)}{N_d\left(\omega_c, \lambda\right)} = b\left(\lambda, \omega_c\right)\Xi\left(\lambda\right), \quad (24)$$

where ψ stands for a dimensionless quantity showing what fraction of the motor power in each mode is not converted into useful power at the transmission output and is closed in the internal circulation.

To assess the energy efficiency of the transmission as a whole, we introduce an integral criterion of circulation losses, which is the average value of the normalized circulation power over the operating speed range.

The integral criterion is determined by the expression

$$\begin{aligned} J\left(\lambda\right) &= \frac{1}{\Delta\omega_c}\int_{\omega_{c,min}}^{\omega_{c,max}} \psi\left(\lambda, \omega_c\right) d\omega_c = \\ &= \frac{1}{\Delta\omega_c}\int_{\omega_{c,min}}^{\omega_{c,max}} b\left(\lambda, \omega_c\right)\Xi\left(\lambda\right) d\omega_c, \end{aligned} \quad (25)$$

where

$$\Delta\omega_c = \omega_{c,\max} - \omega_{c,\min}.$$

The introduced criterion $J(\lambda)$ has the following properties: it is dimensionless due to normalization over the speed range; characterizes the average relative share of circulating power over the entire operating range of motion; allows you to compare different transmission tuning options based on the integral energy indicator, and not on individual modes.

The smaller the value of $J(\lambda)$, the more energetically favourable the corresponding transmission operating mode is from the point of view of circulation losses.

It is the function $J(\lambda)$ that is further used to: search for rational values of the control parameter λ (the ratio of the working volumes of hydraulic machines); com-

parison of alternative options for setting up a power-split hydro-mechanical transmission.

To illustrate the applicability of the proposed mathematical model and to analyse the effect of power circulation on transmission energy performance, an example is considered using parameters typical of a mine diesel locomotive. The relationships presented below serve solely for demonstration purposes and are not intended to be universally representative. The following transmission and engine parameters were adopted for the calculations – rated engine power: $N_{d,nom} = 100$ kW; the number of planetary gear teeth: $Z_s = 30, Z_r = 75, k_z = \frac{Z_s}{Z_r} = 0.4$; gear ratios of branches: $i_s = 0.05$, $i_h = 0.05$; efficiency of branches and planetary gear: $\eta_g = 0.95$, $\eta_h = 0.85$, $\eta_p = 0.92$.

Range of regulation of the ratio of working volumes of hydraulic machines: $\lambda = \frac{q_p}{q_m} \in [0;1]$.

Range of angular velocities of the output link (carrier), corresponding to the operating speeds of the diesel locomotive, rad/s.

$$\omega_{c,\min} = 0.5; \quad \omega_{c,\max} = 10.$$

To define the fraction of engine power transmitted through the mechanical branch, a linear approximation is employed that captures the general trend of power redistribution with variation of the control parameter; specifically, a smoothly increasing function is assumed

$$\beta(\lambda) = 0.25 + 0.6\lambda. \quad (26)$$

This choice means that at $\lambda = 0$ the mechanical branch transmits about 25 % of the motor power; at $\lambda = 1$ – about 85 %, the rest is closed through the hydraulic circuit.

This dependence qualitatively reflects the practical trend: during the transition to transport modes of operation, the share of mechanical power transmission increases, while the contribution of the hydraulic branch decreases.

The circulation of power in a power-split hydro-mechanical transmission is complex and depends both on the speed mode of the planetary gear set and on the transmission settings. To demonstrate the capabilities of the model, the power circulation coefficient is approximated as a product of two factors – the dependence on speed is described by formula (27) and the dependence on the control parameter by formula (28).

$$v_{rel} = \frac{\omega_c}{\omega_{c,\max}}; \quad b_v(v_{rel}) = \exp\left(-\frac{(v_{rel} - v_0)^2}{2\sigma_v^2}\right), \quad (27)$$

where in the calculations it is accepted $v_0 = 0.4$, $\sigma_v = 0.2$.

This form of the function corresponds to maximum circulation effects in the middle part of the operating speed range, which is consistent with qualitative observations for power-split HMT.

The dependence of the circulation coefficient on the parameter λ is given by the expression

$$b_\lambda(\lambda) = b_0[1 - \beta(\lambda)][1 + \alpha(\lambda - \lambda_0)^2], \quad (28)$$

where b_0 stands for basic level of circulation (in calculations 0.3); $\lambda_0 = 0.7$ is the value of the parameter in the vicinity of which circulation is minimal; $\alpha > 0$ is coefficient of "steepness" of circulation growth when deviating from λ_0; the multiplier $[1 - \beta(\lambda)]$ takes into account that with an almost completely mechanical flow $\beta \to 1$, the circulation in the hydromechanical part decreases.

The final expression for the power circulation coefficient is taken as

$$b(\lambda, \omega_c) = \min\{0.9, b_v(\omega_c) b_\lambda(\lambda)\}. \quad (29)$$

The 0.9 limit was introduced to exclude unstable and physically impossible modes, in which the circulating power becomes commensurate with the input power, and there is practically no useful energy transfer to the transmission output.

To analyse the distribution of circulation losses in the space of transmission parameters, the relative circulation power is used, determined according to (24). The numerical analysis was carried out on a uniform computational grid: the control parameter λ was varied from 0 to 1 with an increment of 0.02–0.05, while the angular velocity of the output link ω_c was varied from 0.5 to 10 rad/s with an increment of approximately 0.1 rad/s.

Results. The two-dimensional map (Fig. 1) of the distribution of the relative circulating power $\psi(\lambda, \omega_c)$ constructed based on the calculation results allows us to identify the characteristic patterns of circulation processes in the transmission

As can be seen, the maximum values of the relative circulating power are observed in the region of small values of the parameter λ (0–0.3) at average speeds of the output link ($\omega_c = 3$–5 rad/s). This corresponds to modes with a high proportion of the hydraulic branch, in which intensive power circulation occurs in the planetary summing mechanism.

As the parameter λ increases, the relative circulating power decreases monotonically over the entire speed range. In the region $\lambda \gtrsim 0.6$–0.7λ, circulation losses become insignificant and practically disappear for all considered values of ω_c.

Thus, the distribution map $\psi(\lambda, \omega_c)$ clearly demonstrates that the highest circulation load is typical for modes with a predominance of the hydraulic branch, while an increase in the share of the mechanical path of power transmission leads to a consistent decrease in circulation losses. The obtained result emphasizes the energy advantage of modes with a high share of mechanical transmission within the framework of the model under consideration.

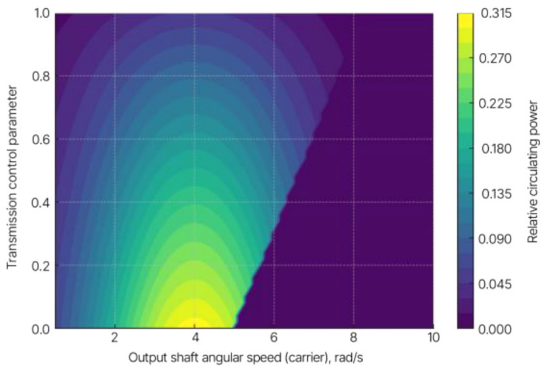

Fig. 1. Distribution of relative circulating power as a function of output speed and transmission control parameter

It should be noted that the zero values of the relative circulating power and overall efficiency observed in certain sections of the calculated dependencies and maps do not correspond to the physical mode of "zero efficiency" of the transmission. These areas are determined by the restrictions on permissible operating modes adopted in the model.

In particular, when the angular velocity of the motor goes beyond the range in which the approximation of its external characteristic remains physically correct, the motor torque in the model is assumed to be zero. Similarly, the introduced limitation on the circulation coefficient is intended to eliminate modes with excessive internal power circulation, in which useful energy transfer becomes inefficient and unstable.

Thus, areas with zero values in the graphs should be interpreted as unconsidered modes that are beyond the applicability of the adopted model, and not as actual operating states of the transmission.

To evaluate the effect of circulation processes on transmission energy efficiency, the variation of the overall efficiency defined by expression (23) is examined. Within the framework of numerical modelling, the dependences $\eta_{tr}(\omega_c)$ were constructed (Fig. 2) for several fixed values of the control parameter λ (in particular, $\lambda = 0$; 0.3; 0.7).

The obtained curves indicate that the parameter λ substantially affects both the absolute efficiency level and the character of its variation with output speed.

The analysis of the obtained relationships indicates that the parameter λ significantly influences both the magnitude of transmission efficiency and the pattern of its variation with the output link speed.

At $\lambda = 0$, corresponding to the dominance of the hydraulic branch, the transmission efficiency remains low throughout the entire speed range, while in the middle region ω_c there is a pronounced dip, coinciding with the zone of maximum power circulation.

At $\lambda = 0.3$, the average level of efficiency increases, but the shape of the curve is still characterized by a noticeable decrease in efficiency in the region of average speeds, which indicates the continuing influence of circulation processes.

The most favourable nature of the dependence $\eta_{tr}(\omega_c)$ is observed at $\lambda = 0.7$: the depth of the dip in the middle part of the range is minimal, and the level of efficiency remains high both at small and at large values of ω_c.

At $\lambda = 1$, the transmission efficiency reaches its maximum values in a wide range of speeds, and the shape of the curve becomes the smoothest, which corresponds to the minimum participation of the hydraulic branch and the lowest level of circulation losses within the framework of the model under consideration.

For a generalized quantitative assessment of circulation losses over the entire operating speed range of the transmission output link, the integral criterion $J(\lambda)$, defined by expression (25), is used. The calculation of the criterion was carried out by the method of numerical integration over the angular velocity of the carrier ω_c using the trapezoidal method on the same computational grid as when constructing the relative circulation power map. The analysis of the dependence (Fig. 3) of the integral criterion $J(\lambda)$ shows a pronounced tendency for its decrease with an increase in the control parameter λ, which indicates a decrease in the relative circulating power with an increase in the share of the mechanical branch in power transmission. The absence of an internal extremum within the considered range of parameters is due to the accepted approximations of power distribution and circulation coefficient and reflects the overall enrgy gain during the transition to modes with a predominance of mechanical transmission.

The function $J(\lambda)$ decreases monotonically over the entire considered range of the parameter $\lambda = q_p/q_m$. The integral criterion attains its highest values at low λ, corresponding to operating modes characterized by a predominant hydraulic branch and pronounced power circulation across the entire speed range. As λ increases, the integral value of the circulating power consistently decreases, which is associated with a decrease in the share of power transmitted through the hydraulic circuit and a corresponding decrease in the total circulation losses. In the region $\lambda = 1$, the value of the criterion $J(\lambda)$ reaches a minimum level, which corresponds to modes with the dominance of the mechanical branch and minimal participation of hydraulics in power transmission.

Thus, within the framework of the adopted model, the integral criterion $J(\lambda)$ indicates a tendency for circulation losses to decrease with increasing parameter λ. The obtained result emphasizes the energy advantage of modes with a high proportion of mechanical power transmission path and can be used in a preliminary assessment of the energy efficiency of power-split hydromechanical transmissions.

Discussion of results. The performed modelling confirms that the circulation component of power in the summing planetary mechanism is an independent factor

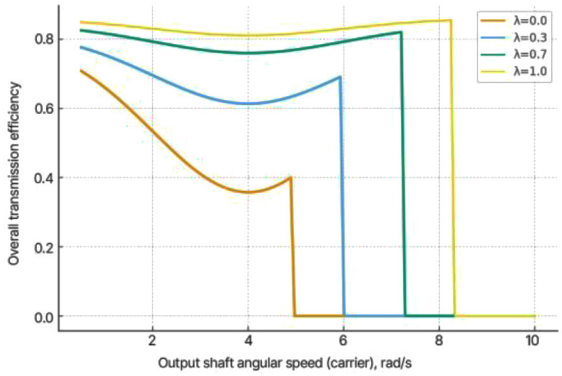

Fig. 2. Transmission efficiency versus output speed for different values of the control parameter

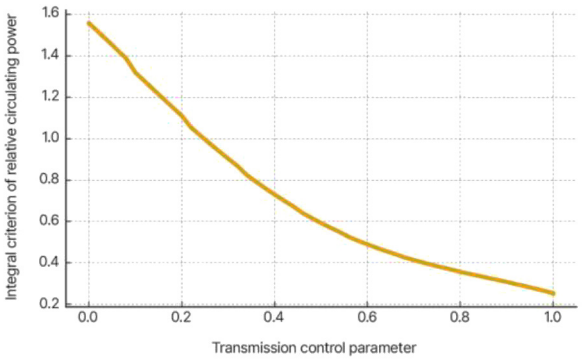

Fig. 3. Integral circulating loss criterion as a function of the transmission control parameter

determining the energy behaviour of a power-split HMT. The resulting distribution map of the relative circulation power shows that the highest circulation load is formed in modes of low values of the control parameter λ, corresponding to an increased share of the hydraulic branch. This is consistent with the physical meaning of the split-power architecture: with increasing participation of the hydrostatic circuit, the probability of closing part of the energy into the internal circulation circuits in the planetary gear increases, which leads to additional losses and heat generation.

The constructed dependences of the overall efficiency demonstrate that the influence of circulation is manifested not only in a decrease in the average level of efficiency, but also in the formation of characteristic "dips" in the area of average speeds, coinciding with the zones of maximum values of the relative circulation power. It should be emphasized that the boundaries of the working areas in the graphs are determined not only by the properties of the transmission, but also by the assumptions of the motor model and restrictions on the circulation coefficient. In particular, when the angular velocity of the motor goes beyond the effective range, the motor torque in the model is assumed to be equal to zero, which corresponds to the physical impossibility of stable operation of the power plant in this mode. Similarly, the introduced limitation on the circulation coefficient is intended to exclude modes in which the circulating power becomes commensurate with the supplied one, and the useful energy transfer practically disappears. Therefore, areas where the calculated curves/maps take zero values should be interpreted as not considered (non-physical or outside the accepted limits), and not as real "zero efficiency".

The integral criterion $J(\lambda)$, calculated over the operating speed range, allows us to move from a point estimate of efficiency to a generalized energy characteristic of transmission tuning. Within the framework of the selected approximations of power distribution and circulation coefficient, there is a pronounced tendency for the integral circulation losses to decrease with increasing λ, that is, with an increase in the share of the mechanical branch. This result reflects the internal logic of split-power systems: the mechanical transmission path has a higher efficiency, and the reduced participation of the hydraulic circuit reduces circulation effects and corresponding losses. At the same time, the quantitative values of $J(\lambda)$ and the shape of the dependencies are sensitive to the selected parameterizations and the power distribution function between the branches, which indicates the need for further identification of the model based on the experimental data of a specific transmission.

Limitations of the work include consideration of steady-state modes without considering dynamics, constant values of efficiency of branches and planetary gear, as well as the use of approximations to describe power distribution and circulation. Despite this, the proposed approach is useful as a tool for preliminary assessment and comparative analysis of HMT settings, allowing one to identify energetically unfavourable areas of regimes and formulate requirements for the rational choice of hydraulic machine parameters and control algorithms.

Conclusions.

1. A mathematical model of a split-power hydromechanical transmission with a summing planetary gear has been developed, incorporating the kinematics of the mechanical and hydraulic branches, the power distribution between them, and the circulating power component within the planetary mechanism.

2. The concept of normalized (relative) circulating power is introduced and the integral criterion ***J*** is proposed, which is the average value of the relative circulating power over the operating speed range of the output link. This ensures a transition from a point estimate to an integral analysis of the energy behaviour of the transmission.

3. Based on the results of a numerical experiment, it is shown that the maximum values of the relative circulation power are characteristic of the region of small λ and average velocities of the output link, which corresponds to modes with an increased share of the hydraulic branch and intensive circulation in the planetary gear.

4. It has been established that an increase in λ leads to a decrease in the circulation component and an increase in energy efficiency, which is manifested in an increase in the level of efficiency and a smoothing of the dependence $\eta(\omega_c)$ when approaching the dominance modes of the mechanical branch.

5. It is shown that the integral criterion $J(\lambda)$ within the framework of the accepted approximations indicates a tendency for a consistent decrease in circulation losses with increasing λ; the criterion can be used for preliminary selection of rational control parameters and comparison of transmission tuning options.

It is advisable to associate further development of the proposed approach with expanding the model by considering unsteady and transient operating modes of the transmission, including acceleration, braking and changes in external load, as well as with the inclusion of inertial properties of rotating masses and the dynamics of the hydraulic system. It is promising to derive the circulating power directly from the balances of the torques and powers of the planetary gear units and refine the loss models considering pressure, rotation speed and thermal state. The implementation of these directions will make it possible to move from mode-energy analysis to a more complete dynamic model of a power-split HMT, focused on the tasks of design, optimization and assessment of the energy efficiency of traction machines in real operating conditions.

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Інтегральний підхід до оцінювання енергетичних втрат у процесі руху тягової машини з гідромеханічною трансмісією

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Мета. Розроблення інтегрального підходу до оцінювання енергетичних втрат, що виникають у процесі руху тягової машини із двопотоковою гідрооб'ємно-механічною трансмісією, з урахуванням циркуляції потужності у планетарному сумуючому механізмі.

Методика. Для досягнення поставленої мети розроблена теоретична модель двопотокової гідрооб'ємно-механічної трансмісії. У моделі враховані кінематичні співвідношення планетарного ряду, розподіл потужності між гілками трансмісії, а також втрати, пов'язані із внутрішньою циркуляцією потужності. Для кількісного оцінювання енергетичних втрат уведено інтегральний показник, що визначається на робочому діапазоні швидкостей руху тягової машини.

Результати. У результаті моделювання встановлено, що величина циркуляції потужності й пов'язані з нею енергетичні втрати істотно залежать від параметрів регулювання гідрооб'ємної передачі та швидкісного режиму руху машини. Виявлено, що в низці експлуатаційних режимів циркуляційна потужність може досягати значних значень, що призводить до зниження загального коефіцієнта корисної дії трансмісії. Отримані залежності, що дають змогу виявити області раціональних параметрів, за яких інтегральні енергетичні втрати є мінімальними.

Наукова новизна. Полягає у формалізації циркуляції потужності як самостійної енергетичної складової процесу руху тягової машини, а також у розробленні інтегрального підходу до оцінювання пов'язаних із нею втрат. Запропонований підхід дає змогу перейти від точкового оцінювання ефективності трансмісії до аналізу її енергетичної поведінки в усьому діапазоні робочих швидкостей.

Практична значимість. Отримані результати можуть бути використані під час проєктування й налаштування гідрооб'ємно-механічних трансмісій тягових машин з метою зниження енергетичних втрат, зменшення теплової навантаженості планетарних механізмів і підвищення ресурсу елементів трансмісії. Інтегральний підхід до оцінювання втрат може бути застосований для обґрунтування вибору параметрів гідромашин і режимів роботи трансмісії в реальних умовах експлуатації.

Ключові слова: *гідрооб'ємно-механічна трансмісія, циркуляція потужності, енергетичні втрати, планетарний механізм, дизелевоз*

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