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L. Shumlianska[*1,2],
orcid.org/0000-0003-0234-7916,
O. Kozionova[3],
orcid.org/0000-0002-2563-8719,
O. Topoliuk[2],
orcid.org/0009-0007-5290-4692,
V. Vilarrasa[1],
orcid.org/0000-0003-1169-4469,
O. Tripil`ska[2],
orcid.org/0009-0002-2457-2671

1 – Global Change Research Group (GCRG), IMEDEA, CSIC-UIB, Esporles, Kingdom of Spain
2 – Institute of geophysics by S. I. Subbotin name, National Academy of Sciences of Ukraine, Kyiv, Ukraine
3 – Taras Shevchenko National University of Kyiv, Kyiv, Ukraine
[*] Corresponding author e-mail: lashum@ukr.net

ASSESSMENT OF CORRECTNESS CONDITIONS IN KINEMATIC SEISMIC TOMOGRAPHY: UNCERTAINTY CALCULATION AND GRID SIZE APPROXIMATION

Purpose. To justify and quantitatively evaluate the characteristic dimensions of regions L within which the linearization of the eikonal equation has a correct solution in the context of kinematic ray tomography based on the principles of geometrical optics.

Methodology. The authors use the theoretical foundations for determining the correctness of solving the seismic problem of the kinematic ray tomography method based on Taylor approximation combined with regularization (the Geyko's method). To assess the characteristic dimensions of the regions, model seismic profiles, including PANCAKE, as well as global mantle tomography data, are applied. The analysis was carried out using models of the velocity structure of the Earth's crust and mantle in the Carpathian region, taking into account the main tectonic units.

Findings. The characteristic dimensions of the linearization region, which determine the resolution of the method, vary significantly – from ≈ 0.2 km in the crust to ≈100 km in the mantle. It was determined that these sizes mainly depend on the geometry of seismic rays and the velocity structure of the medium. The main factor influencing the size of region L was found to be the size of the time spline window selected when forming one-dimensional travel-time curves in the common midpoint format. It was established that the errors of the kinematic tomography method are errors in estimating the depth of penetration of refracted rays. The paper shows how to calculate these errors and use them to assess the accuracy of the kinematic tomography method in the format of seismic velocities.

Originality. Quantitative criteria are proposed for the applicability of the linearized approach to solving the eikonal equation in heterogeneous media within the framework of the kinematic method using Taylor approximation. For the first time, a survey design network for regional tomography by the kinematic method with the possibility of setting the resolution was calculated. The effect of errors on the method's results is demonstrated.

Practical value. The study results have important applied significance for optimizing the configurations of seismic observation networks. They allow for a more substantiated parameterization when constructing kinematic models, ensuring a balance between resolution and stability of solutions. The methodology makes it possible to design kinematic tomography surveys using natural sources of seismic waves with predictability of results at a level comparable to deep seismic sounding methods, despite the uneven distribution of sources and receivers.

Keywords: *kinematic method, seismic tomography, seismic profiles*

Introduction. Ray-based seismic tomography is one of the main geophysical methods for studying the Earth's internal structure at various scales, ranging from regional to local studies. Despite the fact that the wavefield contains incomparably more information than the travel times of seismic waves used in ray tomography, waveform inversion has not yet been able to replace ray methods [1]. This is due to the considerably greater challenges, both in principle and computationally, which, in particular, make the result of waveform inversion dependent on the accuracy of the initial model selection [2]. And it is precisely ray-based seismic tomography that remains the most important tool for creating such models, which underscores its practical value for solving the assigned tasks.

A number of key issues in ray-based seismic tomography remain unresolved, at least for natural seismic wave sources. The main difficulty is primarily related to the uneven distribution of natural seismic wave sources (earthquakes) and seismic stations that record them because, for the correct solution of the ray tomography problem, each point in the medium must be both a source and a receiver of seismic waves [3]. An important issue in ray-based seismic tomography is the assessment of the resolving power of the data and the construction of models that are adequate for this resolving power [4, 5].

An effective assessment of model reliability remains a serious problem in seismic tomography due to the ill-posed nature of the tomographic inverse problem. This leads to the existence of multiple solutions that can be unstable with small changes in initial conditions or data uncertainties, unless regularization is applied [6]. The combination of implicit (e.g., through model parameterization choice) and explicit (e.g., damping and smoothing) regularization, limited knowledge of data noise, uncertainty in prior information, and simplifying assumptions in both forward and inverse steps (e.g., using ray approximation, ignoring anisotropy, linearization) significantly complicates the reliable assessment of model stability.

To assess the spatial resolution of solutions obtained using linear or iterative nonlinear inversion schemes, sensitivity analysis with synthetic models is widely used. One of the most common types of such analysis is the checkerboard resolution test. In this test, a synthetic model includes alternating regions with different wave velocities (or other seismic properties) in 2D or 3D. These tests have several potential drawbacks: 1) they provide only indirect measures of reliability, such as resolution and uncertainty; 2) may give the impression of possible resolution scales; 3) do not provide an accurate representation of structural distortions or smearing caused by data limitations [7]. Originally, these methods were developed to assess spatial resolution in inverse problems where formal resolution is impossible due to the size of the model. They involve creating a heterogeneous synthetic model, based on which the forward problem is solved using the same data coverage as for real observations. The inversion method is then applied to synthetic data to recover the test model. Differences between the true and recovered models form the basis for assessing the reliability of the obtained results.

The methods and approaches to solve the problem of assessing the resolving power and accuracy of ray tomography methods, including kinematic tomography, as described above, highlight the limitations in finding solutions, which are linked to issues of seismic problem accuracy, computational complexities associated with algorithms, creation of synthetic models, and choice of regularization. These factors themselves point to the instability of the problem. This indicates that alternative approaches must be sought to assess the errors and resolving power of the ray tomography method. One possible option is an approach based on the physical laws of geometric seismology.

This work represents, on the one hand, a preparatory stage for constructing a seismotomographic model of the Carpathian region using the kinematic method of Taylor approximation [8]. On the other hand, it addresses the issues of uncertainty in solving the seismic problem of ray tomography at a theoretical level with an original, authorial approach. The Geyko method [8] has several advantages over other kinematic methods of seismic tomography, the main one being a significant improvement in the accuracy of solving the seismic problem due to the way nonlinearities are approximated and the format of constructing one-dimensional hodographs. The assessment of the parameters for solving the seismic tomography problem and uncertainty estimation will be based on two key parameters: 1) the characteristic size of the region, L, within which the wave equation has a correct solution according to Tikhonov's method; 2) the calculation of uncertainties [9] when determining the velocity profile and the depth of maximum penetration of the refracted wave ray. Our approach to the uncertainty estimation will be of interest not only to researchers working with kinematic methods of seismic tomography, but also to a wider group of scientists, as it is based on the physical laws of seismic optics and has not yet been discussed in available literature sources.

Theory. The seismic problem, in general, involves solving the wave equation (the eikonal equation), which describes the spatial distribution of wavefields. In the general case, the wavefield is described by Maxwell's equations, the solutions to which require simplifications. Seismologists are interested in simplifications that provide solutions for wave equations describing the location of fronts and rays of the wavefield from seismic waves. Theoretical solutions for these are represented by the equations of geometric optics – the eikonal equation.

The transition from solutions of Maxwell's equations to the equations of geometric optics is possible by applying a series of simplifications for solving the seismic tomography problem, which belongs to the class of ill-posed problems. So, it is necessary to determine the conditions of correctness according to the Hadamard-Tikhonov approach for solving the ill-posed seismic tomography problem. This procedure is called linearization.

Consider the wave equation in a Cartesian coordinate system in an isotropic medium for a monochromatic wavefield oscillating with angular frequency ω. This allows the equation to be written in scalar form [10] as

$$\nabla^2 u(r) + k_0^2 \tilde{n}^2(r,\omega) u(r) = 0, \qquad (1)$$

where ∇ is the Hamilton operator, $u(r)$ represents any component of the wavefield that depends on distance r, ω is angular frequency; $\tilde{n}(r,\omega)$ is the complex refractive index of the medium, in this case it is a slowly changing function of distance r, $k_0 = \omega/c$, where c is the speed of light in a vacuum.

In a medium with a uniform refractive index $n(\omega)$, the solutions of Maxwell's equations are plane waves. For a homogeneous medium, it becomes possible to describe the field in the first approximation by "local" plane waves of the form

$$E(r, t) = E_0(r) \exp[ik_0 S(r) + i\omega t], \qquad (2)$$

where $E_0(r)$ is a slowly changing function, $S(r)$ is an arbitrary coordinate function of r and t is time. For a homogeneous environment, $E_0 = \text{const}$, $S = ns^*r$. If $k_0 \to \infty$, then in the solutions of equation (2), the term with $1/k_0$ can be neglected, and after some transformations, we arrive at the eikonal equation

$$(\nabla S)^2 = n^2(r). \qquad (3)$$

Function $S(r)$, so-called eikonal, should be chosen in such a way that it satisfies equation (3). At finite values of ω, neglecting the terms with $1/k_0$ is not possible as the error associated with such an approximation when choosing the field in the form of equation (2) is not known in advance.

For plane waves in a homogeneous medium, the amplitude

$$A(r) = |\mathrm{u(r)}| e^{i\varphi(r)}, \qquad (4)$$

changes little over a distance on the order of the wavelength.

Luneburg-Klein asymptotic series is as

$$u(r) \sim e^{-ik_0 S(r)} \sum_{m=0}^{\infty} \frac{A_m(r)}{(-ik_0)^m}. \qquad (5)$$

The asymptotic series of equation (5) is also referred to as the representation of geometric optics. Compared to traditional geometric optics, equation (5) provides a more complete description of wave propagation.

If the series in Equation (5) converges for large k_0, it represents a Taylor series expansion in terms of the wave number and is an exact solution to equation (1).

In general, tomographic methods can be divided into two groups. The first group includes classical linearization methods, based on the idea of linearizing multidimensional inverse kinematic seismic problems. These methods require specifying an initial reference one-dimensional model, and then the solution is sought as the correction to this model. The second group is based on linearizing the tomographic problem by constructing an approximation of the three-dimensional velocity model, regardless of the possible one-dimensional reference model. The method proposed by Geyko [8] belongs to this group. Its foundation is the Taylor approximation of the eikonal equation and the wave equation.

Taylor approximation of nonlinearity – Geyko [8]. Let the refracted wave propagate from the source $x^0 \in S$ to a point $x^1 \in S$ (or vice versa) with slowness p along $\gamma(x^0, x^1)$. Let us denote by $t(x^0, x^1)$ the signal transmission time and through $U(x^0, x^1, \tau)$ – its seismogram. Let us introduce new variables $\xi = (\xi_1, \xi_2)$, X and φ as

$$\xi_1 = \frac{x_1^0 + x_1^1}{2}; \quad \xi_2 = \frac{x_2^0 + x_2^1}{2};$$
$$X = \sqrt{(x_1^1 - x_1^0) + (x_2^1 - x_2^0)^2}; \qquad (6)$$
$$\varphi = \arctan\frac{x_2^1 - x_2^0}{x_1^1 - x_1^0}.$$

Functions $U(\xi, X, \varphi, \tau)$ and $t(\xi, X, \varphi)$ are invariants with respect to the relative position of the source and receiver of the signal.

When expanding the functions into a Taylor series

$$\vec{p}^{\,2}\left(x,z\right), \overrightarrow{\cup}\left(\xi, X, \tau\right), \vec{t}\left(\xi, X\right), \qquad (7)$$

with respect to the variables x_1, x_2 and ξ_1, ξ_2 in the neighborhood of the point $\zeta = (\zeta_1, \zeta_2)$; thus, by limiting to only the terms of the zero order series, the Taylor formulas hold: $\vec{p}^{\,2}(x,z) = \vec{p}_\varsigma^{\,2}(z) + R(\vec{p}^{\,2})$, $\overrightarrow{\cup}(\xi, X, \tau) = \overrightarrow{\cup}_\varsigma(X, \tau) + R(\overrightarrow{\cup})$, $\vec{t}(\xi, X) = \vec{t}_\varsigma(X) + R(\vec{t})$, where $R(\vec{p})$, $R(\overrightarrow{\cup})$, $R(\vec{t})$ are remainder terms.

It has been proven that when $\xi \to \varsigma$ in the neighbourhood of the point $\zeta \in S$, the following zero-order Taylor approximations of the wave equation and the eikonal equation are valid

$$\frac{\partial^2 \overrightarrow{\cup}_\varsigma(X,\tau)}{\partial z^2} = \vec{p}_\varsigma^{\,2}(z)\frac{\partial^2 \overrightarrow{\cup}_\varsigma(X,\tau)}{\partial \tau^2}; \qquad (8)$$

$$\left(\frac{\partial \vec{t}_\varsigma(X)}{\partial z}\right)^2 = \vec{p}_\varsigma^{\,2}(z). \qquad (9)$$

Equations (8 and 9) are uniquely solvable for the function $\vec{p}_\xi(z)$, the corresponding seismogram $\overrightarrow{\cup}(X,\tau)$ and the hodograph $\vec{t}_\xi(X)$.

The solution to the seismic tomography problem obtained using this method is an exact lower bound relative to the solution obtained by the classical linearization method, and it imposes fewer constraints on the velocity function. The method is independent of the choice of the initial approximation (one-dimensional reference model) and is Tikhonov-regularized.

The classical linearization of the seismic problem differs from Geyko's linearization [8] in terms of the format of the hodograph and the use of slowness instead of velocity, as in others. The formation of the hodograph (the dependency of the arrival times of the first seismic wave arrivals on distance) in the average point format, rather than along the entire ray, i.e., at the point of maximum ray penetration, allows the hodograph to be formed at the points of maximum ray penetration. Moreover, the ray is considered symmetric, so the point of maximum penetration has coordinates ($X/2$; $T/2$, where X is the distance, and T is time according to the hodograph to the hodograph), and the time field is formed in the average point format.

The principle of slowness is exactly satisfied in the case of a plane or nearly plane wave propagating in a homogeneous or nearly homogeneous medium. This allows for solving the seismic tomography problem by using piecewise-constant velocity functions as a parameter of the medium as one of the approaches.

The fulfilment of the slowness principle follows from the necessity of a small change in the local wave vector in order to solve the wave equation for a plane wave [3] as

$$\left|n(r+L) - n(r)\right| \sim n(r), \qquad (10)$$

where L is defined as the distance over which the change in some quantity occurs ($A(r)$, $k(r)$, $n(r)$) comparable to the quantity itself or changes little over the wavelength in the medium $\tilde{\lambda}(r) = 1/\left|k(r)\right|$ and gives an estimate for L

$$L \sim n/\left|\nabla n\right|, \qquad (11)$$

which defines the region of stability of the linearization for the eikonal equation.

L is related to the wavelength through the local wave vector $k(r)$,

$$\tilde{\lambda}\left|\nabla A\right| \ll \left|A\right|; \qquad (12)$$
$$\tilde{\lambda}\left|\nabla k_j\right| \ll \left|k_j\right|; \qquad (13)$$
$$\tilde{\lambda}(\nabla n) \ll n, \qquad (14)$$

where k_j is component j of the wave vector k; A is amplitude; $\tilde{\lambda} = \lambda/2\pi$; ; $n(r)$ is refractive index.

The dependence of the area size L on the wavelength establishes the sufficient conditions for the applicability of geometric optics, based on the concept of the Fresnel volume [3]. Given that the Fresnel volume defines the region that forms the field at a given point, i.e., it is the localization region of the ray, two conditions for the applicability of geometric optics in the case of a plane or nearly plane wave are formulated: 1) the parameters of the medium, as well as the parameters of the wave (amplitude and phase gradient), should not change significantly across the cross-section of the Fresnel volume; 2) the Fresnel volumes of the rays arriving at the same point should not overlap significantly.

At the same time, the principle of slowness in the Geyko method [8] is exactly satisfied under the assumptions that we are dealing with a plane or nearly plane wave propagating in a homogeneous or nearly homogeneous medium. This allows the use of piecewise-constant velocity functions as a parameter for the medium in solving the seismic tomography problem. It is clear that the applicability of both methods is determined by the wavelengths used and the Fresnel volumes.

Calculation of the uncertainty $\varepsilon_0(q)$ *and characteristic size L*. Burmin [9] notes: "There is a misconception that the solution to the problem of inverting the ray diagram should provide a range of velocity functions, which contain velocity functions that satisfy the observed ray diagram of seismic waves. At the same time, this range is associated with the uncertainty estimation in determining the function $v(z)$. The meaning of these estimates is not entirely clear. The fact is that for any velocity interval on the given segment $[z_1, z_2]$, an infinite number of velocity curves can be constructed that do not correspond to the observed ray diagrams. In this case, any point of this range, by definition, should be a solution to the problem, which, of course, is not the case. Therefore, if a range of possible solutions is provided, it is necessary to define the properties of the velocity functions that fill the given range and to establish an algorithm for constructing this range in such a way that each function in the range is a solution to the inverse problem. At present, such an algorithm does not exist, and such functions are not defined. When estimating the uncertainty in determining the velocity profile, one should also consider the uncertainty in determining the depth of maximum penetration of the refracted wave ray".

$$\varepsilon_0(q) \le \varepsilon(u^*) + \frac{\delta_0}{\pi}\ln\left(\frac{u^*}{q}+\sqrt{\frac{u^{*2}}{q^2}-1}\right). \qquad (15)$$

The value δ_0 is the maximum deviation of the approximating spline from the experimental points of the seismogram, where $\delta_0 = \max\left|\delta x(p)\right| = \max\left|\tilde{x}(p) - x(p)\right|$ corresponds to $\max\left|\tilde{t}(p) - t(p)\right|$ and is defined as $\delta t(p)/p$, where $\delta t(p)$ (the superscript ~ indicates values with uncertainty, without the theoretical value). The function $z(q)$ is the dependence of the ray parameter q on depth z and is defined at each point of the segment $[q_1, q_2]$. $\varepsilon(u^*)$ is the absolute value of the uncertainty in determining z^*, $z^* \le z_m \le z_M$, $x(p, u^*)$ represents the distance to the point where the seismic ray exits, penetrating to a depth $z_m \ge z^*$, dependent on the ray parameter p, $q = u(z_m) \le p \le$ $\le u(z) = u^*$.

Calculation of the characteristic size L. As shown in equation (11), $L \sim n/|\nabla n|$, where n is the refractive index for each layer of the medium, ∇ is the Hamiltonian operator for a three-dimensional Cartesian medium, and, in our case, we use a one-dimensional representation, so the change in the refractive index is considered only for the vertical projection z, i.e., depth.

Study region. The seismic velocity data for the crust used in our study were taken along the profile PANCAKE. Between 2008 and 2011, an international team of scientists conducted research using wide-angle reflection/refraction (WARR) survey along the PANCAKE (PANnonian-CArpathians-Cratonic Europe) profile, which crosses the Pannonian Basin, the ALCAPA microplate, and partially the Tisa-Dacia region along the Mid-Hungarian Tectonic Zone, the Ukrainian Carpathians, the Trans-European Suture Zone (TESZ), and part of the East European Craton (EEC) (Fig. 1). The results of the study, based on modern tech-

Fig. 1. Schematic tectonic map of the research area and profile location PANCAKE (Starostenko, et al., 2013):

PB – Podillia Block; ESB – East Slovakian Basin; HCM – Holy Cross Mountains; KP – Korosten Pluton; MHL – Mid Hungarian Line; NU – Narol Unit; OC – Outer Carpathians; OWC – Outer West Carpathians; PKB – Pieniny Klippen Belt; RŁU – Radom-Łysogóry Unit; RRFZ – Rava Ruska folded zone; TESZ – Trans-European Suture Zone; VB – Volyn Block; VGVR – Vyhorlat-Huta Volcanic ridge; VPPB – Volyn-Polissia Plutonic Belt. Black lines with black triangles represent main trusts, blue areas filled black lines represent Pieniny Klippen Belt, cyan areas represent inner Carpathian–Alpine units, orange areas represent Neogene volcanics. Solid black line represents the edge of East European Craton

nologies and data interpretation, refined the understanding of the lithosphere structure in the region down to a depth of 65 km. These units, ranging in age from the Archean to the Quaternary period, differ in geological origin and geodynamic history [11].

The field campaign included 14 explosive sources (3 in Hungary and 11 in Ukraine) and 261 stations with single-component geophones, spaced every 2.5 km. The used seismic phases include refractions from sedimentary layers (P_{sed}), the upper/middle crystalline crust (P_g), and the upper mantle (P_n). The seismic data show a variable wavefield, reflecting differences in the structure of tectonic units.

Within the Eastern European Craton (EEC) (from 650 to 340-km depth), the following structural elements were intersected: the southwestern slope of the Ukrainian Shield, the Volyn-Polissia Basin, the Volyn-Podillia Monocline, and the Lviv Basin. The profile then passes through the Eastern Carpathians (160–340 km), divided into two units: the Carpathian Foredeep (290–340 km), above the Trans-European Suture Zone (TESZ), and the Outer Carpathians (200–290 km). At the southwestern end, the profile crosses the Pannonian Basin.

The TESZ (Trans-European Suture Zone) on the southwestern boundary of the Eastern European Craton (EEC) includes units of the Earth's crust that separate the "ancient" Europe from the Variscan and Alpine orogens. The Carpathian Foredeep consists of Neogene molasses with an autochthonous Miocene sedimentary unit on the Paleozoic-Mesozoic cover of the EEC and TESZ, as well as an internal allochthonous zone. The Outer Carpathians are fold structures thrust over the Carpathian Foredeep (Kruglov S.S., et al., 1985).

The origin of the Carpathian-Pannonian region remains controversial. Some authors believe that it formed during the Tertiary period due to the subduction of oceanic lithosphere beneath the microplates ALCAPA and Tisa-Dacia (Ustaszewski, et al. 2014), while others associate it with rifting or gravitational instability (Starostenko & Gintov, 2018) [12]. The Pannonian Basin is covered with Miocene-Pliocene deposits of varying thickness, and the pre-Tertiary basement of the ALCAPA microplate in the northeast consists of Mesozoic carbonaceous deposits [13].

For the mantle, data for P-wave velocities are taken from the solution of the seismic problem using the kinematic method of Taylor approximation [14]. The seismotomographic model is taken for the mantle area beneath Ukraine, within the depth range of 50–2,700 km [14].

Methods. The calculation of uncertainties and characteristic sizes L is carried out as follows. At the first stage, the uncertainties $\varepsilon_0(q)$ in determining the depth of maximum penetration of the refracted wave ray are estimated for different input data on $\varepsilon(u^*)$ – the absolute value of the uncertainty in determining the slowness p –, for values of $\varepsilon(u^*) = 0.1, 0.2, 1, 2$ s/km; δ_0 – the maximum deviation of the approximating spline from the experimental points of the travel-time curve, $\delta_0 = 0.1, 0.2, 1, 2$. Since we do not know the accuracy of the determination or the deviation of the experimental value from the calculated value, such as the travel time T of the seismic wave over the distance from the source to the point of exit of the seismic ray X, we take values for the calculation of the uncertainty purely hypothetically, with a large spread, in order to examine the influence of these parameters on the uncertainties over a wide range and identify the parameter that has the strongest influence on the uncertainty.

Results. Fig. 2 shows the uncertainties in determining the depth of maximum penetration of rays for six selected integral reference velocity columns, constructed based on the PANCAKE profile data and the seismotomographic model for the mantle obtained using the [8] method. The velocity columns are presented in Table 1. Six segments are selected from the PANCAKE profile, corresponding to tectonic structures, and it should be noted that simplifications are made for the velocity columns in the Earth's crust. The following columns are selected based on tectonic structures: EEPVPL (East European Platform, Volyn-Podillya Block); EEPLPP (East European Platform, Lviv Paleozoic Basin); ECARPP (Pericarpathian Basin; ECARPSK (Eastern Carpathians); ALKAPA (Alkapa); TISIA (Tisa-Dacia).

The calculation of the characteristic size L is carried out with different uncertainty values $\varepsilon(u^*)$, d_0 for various values based on the given integral velocity columns, selected as reference models, constructed from the PANCAKE profile and the seismotomographic model of the mantle (Fig. 3).

The refractive index is obtained from Snell's law, taking into account that for air the refractive index is

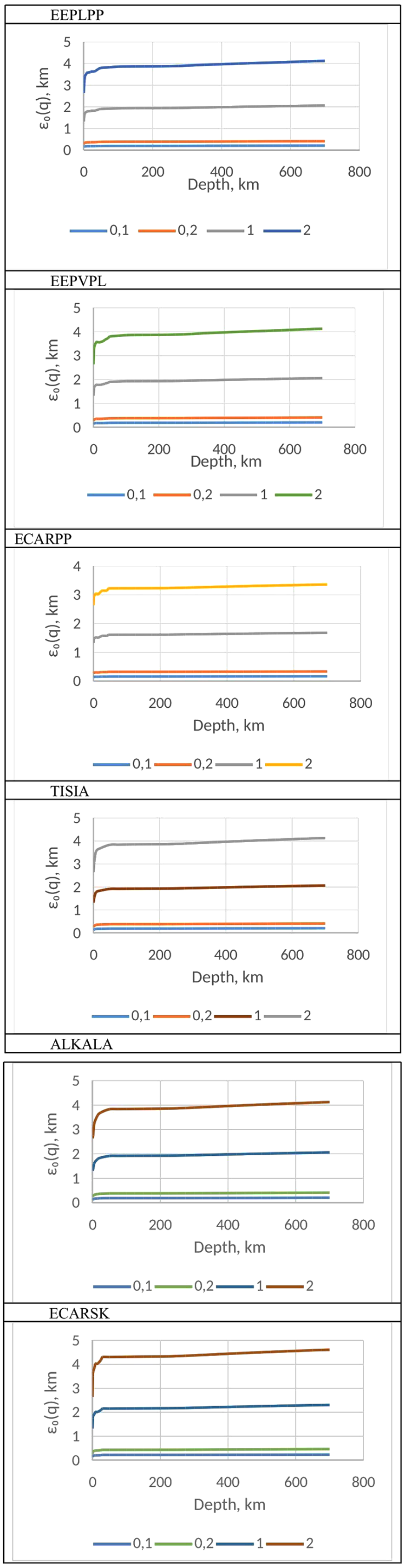

Fig. 2. Uncertainty $\varepsilon_0(q)$ estimates in determining the depth of maximum ray penetration for different values of seismic wave velocity for six selected integral reference velocity columns, constructed based on the PANCAKE profile data and the seismotomographic model for the mantle for $\varepsilon(u^*) = 0.1;\ 0.2;\ 1.0;\ 2.0$ s/km, $\delta t = 0.1;\ 0.2;\ 1.0;\ 2.0$ s, $d_0 = 0.1;\ 0.2;\ 1.0;\ 3.0$ km

Table 1

Seismic velocity V_p values for integral velocity models of the crust and upper mantle, chosen as reference for 6 tectonic structures in the Carpathian region. Data for the crust is from [11], data for P-wave is from [14]

EEPLPP		EEPVPL		ECARPP	
H, km	V_p, km/s	H, km	V_p, km/s	H, km	V_p, km/s
0	2.4	0	2.4	0	2.4
1	3.1	1	3.1	1	4.35
2	4.5	2	4.5	2	4.35
3	4.5	5	5.15	6	4.65
9	6.2	9	6.2	10	5.45
10	6.27	10	6.23	14	5.55
19	6.27	15	6.27	22	5.29
20	6.6	20	6.09	25	6.4
30	6.6	30	6.25	27	6.7
34	6.7	35	6.7	40	6.93
41	6.93	45	6.93	45	6.93
50	7.769	50	7.769	50	8.017
75	7.848	75	7.848	75	8.086
100	8.215	100	8.215	100	8.124
125	8.278	125	8.278	125	8.151
150	8.31	150	8.31	150	8.174
175	8.32	175	8.32	175	8.195
200	8.333	200	8.333	200	8.217
225	8.354	225	8.354	225	8.245
250	8.389	250	8.389	250	8.288
275	8.446	275	8.446	275	8.407
300	8.545	300	8.545	300	8.519
325	8.709	325	8.709	325	8.644
350	8.884	350	8.884	350	8.778
375	9.011	375	9.011	375	8.916
400	9.103	400	9.103	400	9.055
425	9.236	425	9.236	425	9.191
450	9.386	450	9.386	450	9.325
475	9.503	475	9.503	475	9.455
500	9.608	500	9.608	500	9.583
525	9.711	525	9.711	525	9.709
550	9.817	550	9.817	550	9.833
575	9.932	575	9.932	575	9.957
600	10.057	600	10.057	600	10.08

End of Table 1

625	10.19	625	10.19	625	10.202
650	10.326	650	10.326	650	10.326
675	10.458	675	10.458	675	10.45
700	10.583	700	10.583	700	10.576
0	2.4	0	2.4	0	2.4
1	3.1	2	2.4	2.5	2.4
2	4.35	5	3.1	8	5.95
6	4.65	15	6.26	24	6.26
9	6.2	25	6.26	50	8.017
12	6.2	50	8.017	75	8.086
16	6.2	75	8.086	100	8.124
25	6.4	100	8.124	125	8.151
30	8.1	125	8.151	150	8.174
50	8.017	150	8.174	175	8.195
75	8.086	175	8.195	200	8.217
100	8.124	200	8.217	225	8.245
125	8.151	225	8.245	250	8.288
150	8.174	250	8.288	275	8.407
175	8.195	275	8.407	300	8.519
200	8.217	300	8.519	325	8.644
225	8.245	325	8.644	350	8.778
250	8.288	350	8.778	375	8.916
275	8.407	375	8.916	400	9.055
300	8.519	400	9.055	425	9.191
325	8.644	425	9.191	450	9.325
350	8.778	450	9.325	475	9.455
375	8.916	475	9.455	500	9.583
400	9.055	500	9.583	525	9.709
425	9.191	525	9.709	550	9.833
450	9.325	550	9.833	575	9.957
475	9.455	575	9.957	600	10.08
500	9.583	600	10.08	625	10.202
525	9.709	625	10.202	650	10.326
550	9.833	650	10.326	675	10.45
575	9.957	675	10.45	700	10.576
600	10.08	700	10.576	–	–
625	10.202	–	–	–	–
650	10.326	–	–	–	–
675	10.45	–	–	–	–
700	10.576	–	–	–	–

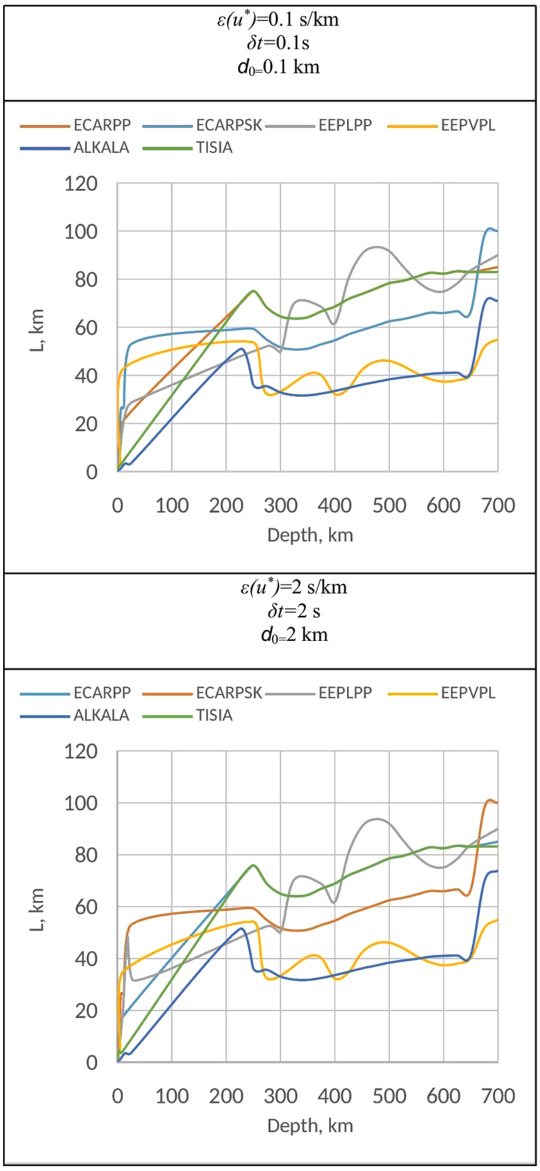

Fig. 3. The results for the calculation of the characteristic sizes L for the six reference velocity models

equal to 1. The results for the calculation of the characteristic sizes L for the six reference velocity models are shown in Fig. 3. As we can see, the characteristic sizes L practically do not change depending on the magnitude of the uncertainties, so we will only provide two figures.

Discussion. Before proceeding to the discussion, it is necessary to clearly distinguish between two concepts: uncertainty and the resolving power of the method. Solving ray-based seismic tomography problems, in addition to an overall assessment of the method's applicability, requires evaluating the uncertainties in the input data. The primary uncertainty in the methods used is the approximate fulfillment of the necessary and sufficient conditions of geometric optics. First and foremost, this involves replacing possible nonlinear wave processes with linear ones, and the method's exact applicability is restricted to homogeneous media, with flat or nearly flat waves. A wave process is defined as linear if the superposition principle holds for it, which is fundamental in geometric optics.

The uncertainty in solving ray-based seismic tomography problems has two components [3]: the primary uncertainty is the evaluation of the methods used when approximating (linearizing) the nonlinear seismic process to satisfy the necessary and sufficient conditions of geometric optics; the secondary uncertainties arise during the processing of seismic records at the stations.

This is a conditional division, since the first and second uncertainties are clearly interrelated, and the resulting uncertainty is not simply their sum. This is demonstrated in our study. The uncertainty we evaluate in our study $\varepsilon_0(q)$ i.e., the uncertainty in determining the depth of the maximum penetration of the refracted wave, as shown by equation (15), depends on $\varepsilon(u^*)$ – the absolute value of the uncertainty in determining the slowness p, and this dependence is linear. Equation (15) also shows the dependence of the sought uncertainty on the values of slowness at the upper and lower boundaries of the depth interval, as well as the size of the temporal window of the spline approximation when forming the travel-time curve. The uncertainty in determining slowness depends on the uncertainties that arise during the processing of seismic records at the stations, which in turn also contain random observational uncertainties associated with actual observations, systematic uncertainties related to hypocenter coordinate determinations, and deviations related to departures from the symmetrical Earth model. As noted by Tsvetkova [3], "to assess the uncertainties of the input data, it is desirable to have minimal wavelengths for each of the records used".

As we can see, uncertainty estimation is a multi-level task when solving ray-based seismic tomography problems. It is difficult to imagine conditions where all measurement uncertainties and nonlinear approximations of various levels are known. Therefore, it is important to understand which uncertainties have the greatest influence on the result.

In our study, we are forced to use conditional uncertainties in the determination of slowness, as their true values are unknown. We select values of $\varepsilon(u^*)$ over a wide range (from 0.1 to 2.0) to examine their influence on the uncertainty $\varepsilon_0(q)$ in determining the depth of maximum penetration of the refracted wave. Knowing the layer thickness and the velocities at the upper and lower boundaries, we can estimate the minimum and maximum deviations in velocity at the base of the layer, given the $\varepsilon(u^*)$ values calculated during the analysis, as $\varepsilon(v) = \varepsilon_0(u^*)/((v_2 - v_1)/h)$, where it is assumed that the velocity gradient is preserved when moving the lower boundary of the layer both upward and downward, accounting for the uncertainty of $\pm\varepsilon(u^*)$, and h represents the layer thickness.

In general, the curves of the uncertainties $\varepsilon_0(q)$ that we calculate represent a normal distribution (Gaussian distribution), meaning a bell-shaped curve with a peak at the center (or, in our case, close to it). As it is known, in a normal distribution of errors, the expected value is zero, meaning that the observation results correspond to or are close to the true values of the physical quantity. Of course, this statement is only valid when systematic uncertainties are excluded. For this, data obtained from different seismic stations, over different years, and from sources with varying intensities should be used. Thus, the probability of systematic uncertainty is practically reduced to zero.

Knowing the uncertainties, $\pm\varepsilon_0(q)$ and $\pm\varepsilon(v)$, it becomes possible to determine the conditions under which the necessary and sufficient conditions of geometric optics are met to define the conditions for approximating the nonlinear seismic process. Specifically, it allows us to determine the characteristic size of region L, within which the inversion of the seismogram, collected in the format of the midpoint ($X/2$; $T/2$), occurs with the specified accuracy $\varepsilon(u^*)$.

On the other hand, the characteristic size of region L determines the resolution capability of the method, as its size corresponds to the edge length of the cube (i.e., both horizontal and vertical dimensions are fixed) of the geophysical medium that makes up the specified volume of the study area.

A similar concept exists in the method of reflected waves, where the resolution capability is divided into horizontal and vertical components. The horizontal resolution capability in seismic exploration is approximately estimated by the formula: $L \geq 2h\lambda$, where h is the depth to the boundary, and λ is the wavelength. From this formula, it follows that the greater the depth, the lower the resolution capability. The vertical resolution capability in seismic exploration is estimated by the formula $L \geq \lambda 4$. It is clear that the shorter the wavelength, the better the vertical resolution capability. Therefore, by selecting different wavelengths, we can alter the resolution capability. As is well known, the resolution capability is determined by the diameter of the first Fresnel zone, and this relationship is shown in the formulas above.

In our study, we observe a similar situation: with increasing depth, the characteristic size L increases (Fig. 3), which means that the resolution capability decreases and the volume of averaged seismic velocity regions increases. Furthermore, our research shows that the size L depends not only on the velocity structure of the medium, but also on the geometry of the ray (Fig. 3).

The approach presented for calculating the resolution capacity of the method and uncertainties are similar to the Spakman-Nolet [15] method for creating synthetic models for tests evaluating solution stability, where a heterogeneous synthetic model is created. This model is used to solve the forward problem with the same data coverage as the observational dataset, followed by inversion of the synthetic data set in order to reconstruct the test model. The differences between the true and reconstructed models form the basis for assessing the reliability of the solution. Although the "similarity" is largely conditional, our approach has significant differences.

Unlike the Spakman-Nolet method [7], we control the resolution capacity of the method by setting specific uncertainty values, which largely depend on the choice of the time window size for the approximating spline. These uncertainties are independent of the density of points within the time window at the maximum ray penetration depth ($X/2$, $T/2$) and their distribution along the travel-time graph, as in the method of the generalized ray density tensor [7], which quantitatively defines the spatially-dependent azimuthal coverage of rays. Within the parameters we set, the solution is unambiguous. The calculation itself is based on the physical laws of wavefront propagation.

There is another significant advantage of the physical approach we use. In the Spakman-Nolet method [15], the synthetic model assumes a fixed location of the source-receiver pairs, so the calculations are performed with a corresponding matrix. In our approach, the determination of characteristic sizes L – the sizes of the correct solution for the ray problem – and the determination of the temporal window size for the spline approximation of the travel-time curves are necessary and sufficient conditions for the unambiguous solution of the seismic problem within the chosen solution correctness conditions. Moreover, the Geyko method [8] of Taylor approximation for the seismic problem involves constructing the travel-time graph in the format of the central point ($X/2$, $T/2$), as shown in the theoretical section. This format provides a significant advantage in solution accuracy and is Tikhonov-regularized.

As for the uncertainties, it should be noted that in the work by Burmin [9], when estimating the uncertainty in the determination of the velocity profile, the focus is on estimating the uncertainties $\varepsilon_0(q)$(q is the value of the slowness at the lower boundary of the layer) in determining the depth of maximum penetration of the refracted wave ray. The calculation of uncertainties in determining the depth of maximum ray penetration is presented in Fig. 2. As follows from the results, the uncertainty in determining the depth of maximum ray penetration of the refracted wave $\varepsilon_0(q)$ depends on the time window of the approximating splines δ_0 and the uncertainty in determining the ray parameter (velocity) $\varepsilon_0(q)$, which is specified depending on the velocity structure of the study area. By setting the aforementioned parameters, we control the uncertainty $\varepsilon_0(q)$, thereby creating the necessary format for the characteristic sizes of L. As seen from Fig. 3, the size of the regions depends on the velocity structure of the reference model and, to a lesser extent, on the uncertainties.

The uncertainty estimation in determining the depth of maximum ray penetration shows its dependence primarily on the size of the spline approximation window and the assigned uncertainty in the determination of the slowness parameter $\varepsilon_0(q)$. The velocity structure also influences the shape of the resulting uncertainty curves in determining the depth of maximum ray penetration (Fig. 2), but not significantly, as seen from the small variations in the numerical values of the uncertainties for the six reference velocity columns we selected.

If we consider this study as the initial stage for constructing a seismotomographic model of the Carpathian region, then, due to the specific tectonic structure of the Carpathian region – parallel tectonic structures stretched along the Carpathian Mountains, having the sizes of L for different velocity columns, i.e., tectonic structures, we can control the size of the expected uncertainties in determining the depth of maximum ray penetration for different seismic wave velocities.

If we consider the theoretical part, this study can serve as an example of how the characteristic size L, where the seismic problem is solved correctly according to Tikhonov's approach, correlates with the uncertainty estimates in determining the depth of maximum ray penetration for various seismic wave velocities. The study demonstrates that by choosing a certain characteristic size L, we can control the uncertainties of the kinematic seismic tomography method using Taylor approximation, $\varepsilon_0(q)$ and the resolution of the kinematic method of seismic tomography. These patterns will also apply to other kinematic approaches. Furthermore, the relationships between the choice of the temporal window size of the approximating spline, the uncertainties in determining the ray penetration depth, and the characteristic sizes L are shown.

Thus, by setting the parameters for the characteristic dimensions L, we set the initial network for solving the problem of seismic tomography by the kinematic method, and the solution is correct according to Hadamard-Tikhonov. Also, having data on the uncertainties in determining the parameters of earthquakes at seismic stations, it becomes possible to calculate the parameters of the results of the kinematic method of seismic tomography: the resolution of the method, the accuracy of determining the penetration depth of the refracted beam, and also to estimate the uncertainty in determining the seismic velocities (v_p and v_s). This allows us to use natural sources of seismic vibrations, earthquakes, for seismic tomography despite the limitations associated with the unevenness of the observation network and earthquake foci.

Conclusions. Ray tomography relies on the principles of geometric seismology and geometric optics. To accurately solve direct and inverse kinematic seismic problems, it is necessary to meet the required physical conditions. These include the stability of medium and wave parameters across the Fresnel volume cross-section and minimal overlap of Fresnel volumes from different rays reaching the same point.

A key parameter ensuring the fulfillment of these conditions is the characteristic region size L, which defines the domain of correct linearization for the eikonal equation. This region size is essential for constructing the dimensional grid used in solving the kinematic problem, particularly within the framework of the Geyko method that incorporates regularization to stabilize the inversion process.

In this study, characteristic region sizes were estimated for application in ray tomography of the Carpathian region. Reference velocity models were developed based on major tectonic structures, including the East European Platform, the Volyn-Podillia Plate, the Lviv Paleozoic Basin, the Pericarpathian Basin, the Eastern Carpathians, Alkapa, and Tisa. These models served as the basis for evaluating the applicability of the kinematic tomography method using Taylor's approximation in a complex regional geological setting. The results of the study are useful in planning seismic surveys using the Geiko method. The obtained L dimensions show the resolution of the method for the data we use on the velocity structure of the Carpathian region. If it is necessary to increase or decrease the desired resolution when planning seismic work using the kinematic tomography method, the data we obtained on the L dimensions should be used.

The results show that the characteristic region sizes vary significantly depending on depth and location, ranging from about 0.2 kilometers in the crust to around 100 kilometers in the mantle. This variation reflects differences in observation network geometry requirements for imaging the crust versus the mantle. The tomographic grid was modeled as a system of cubes, where all observation points within a cube contribute to a one-dimensional hodograph.

The main factors influencing region size are the velocity structure of the medium and the geometry of seismic ray paths. Uncertainties in estimating the depth of ray penetration were found to be secondary and primarily dependent on the size of the spline approximation time window and the definition of delay parameters.

Ultimately, the characteristic region size determines the achievable resolution in kinematic ray tomography. The ability to estimate this size for a given uncertainty level is critical for the design of seismic observation networks. This makes the kinematic tomography approach using Taylor's approximation comparable in resolution to deep seismic sounding methods, enabling accurate modeling even with natural seismic sources, despite the challenges posed by uneven station and source distribution.

The findings of this study are valuable for configuring observation networks and choosing parameterization strategies in kinematic tomography. They also provide a foundation for future studies on how spline approximation settings affect modeling uncertainty, emphasizing the importance of balancing resolution and stability in seismic imaging.

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Оцінка умов коректності в кінематичній сейсмічній томографії: розрахунок невизначеності й апроксимація розміру сітки

Л. Шумлянська[*1,2], *О. Козіонова*[3], *О. Тополюк*[2], *В. Віларраса*[1], *О. Трипільська*[2]

1 – Дослідницька група глобальних змін (GCRG), IMEDEA, CSIC-UIB, м. Еспорлес, Королівство Іспанія
2 – Інститут геофізики імені Субботіна НАН України, м. Київ, Україна
3 – Київський національний університет імені Тараса Шевченко, м. Київ, Україна
[*] Автор-кореспондент e-mail: lashum@ukr.net

Мета. Обґрунтування й кількісна оцінка характеристичних розмірів областей L, у межах яких лінеаризація рівняння ейконалу має коректне рішення в контексті кінематичної променевої томографії, що базується на принципах геометричної оптики.

Методика. Автори використовують теоретичні засади визначення коректності рішення сейсмічної задачі методу кінематичної променевої томографії на основі апроксимації Тейлора в поєднанні з регуляризацією (метод Гейка). Для оцінки характеристичних розмірів областей застосовані модельні сейсмічні профілі, зокрема PANCAKE, а також дані глобальної томографії мантії. Аналіз проводився із застосуванням моделей швидкісної будови земної кори й мантії Карпатського регіону з урахуванням головних тектонічних одиниць.

Результати. Характеристичні розміри області лінеаризації, що задають роздільну здатність методу, значно варіюються – від ≈ 0,2 км у корі до ≈ 100 км у мантії. Визначено, що ці розміри залежать, переважно, від геометрії сейсмічних променів і швидкісної структури середовища. Основним фактором, що впливає на розмір області L, виявлено розмір вікна часового сплайну, який вибирається при формуванні одномірних годографів у форматі середньої точки. Встановлено, що похибки методу кінематичної томографії є похибками оцінки глибини проникнення рефрагованих променів. Показано, як їх розраховувати й використовувати для оцінки точності методу кінематичної томографії у форматі сейсмічних швидкостей.

Наукова новизна. Запропоновані кількісні критерії придатності лінеаризованого підходу до розв'язання рівняння ейконалу в межах неоднорідних середовищ кінематичного методу Тейлорової апроксимації. Уперше розрахована мережа проєктування робіт для регіональної томографії кінематичним методом із можливістю завдання роздільної здатності. Показано вплив похибок на результати методу.

Практична значимість. Результати дослідження мають важливе прикладне значення для оптимізації конфігурацій сейсмічних мереж спостереження. Вони дозволяють більш обґрунтовано формувати параметризацію при побудові кінематичних моделей, забезпечуючи баланс між роздільною здатністю й стабільністю розв'язків. Методика дозволяє проєктувати роботи методом кінематичної томографії з використанням природних джерел сейсмічних коливань із передбачуваністю в отриманні результатів на рівні методів глибинного сейсмічного зондування, не дивлячись на нерівний розподілом джерел коливань і приймачів.

Ключові слова: *кінематичний метод, сейсмічна томографія, сейсмічний профіль*

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