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IMPLEMENTATION OF A COMPUTATIONAL EXPERIMENT FOR SHOCK INTERACTION OF SPHERICAL BODIES

Purpose. To identify the time-dependent patterns in the kinetic and dynamic parameters of impact-based contact interactions among objects, as derived from a computational experiment to construct relevant approximation relationships for designing technological processes to load spherical bodies, specifically pellets.

Methodology. To determine the temporal laws governing changes in the kinematic and dynamic parameters of contact interactions and to develop the corresponding computational experiment model, a simulation-based approach was employed. This approach relies on solving the Hertz contact problem and modelling the relative motion of objects within a homogeneous coordinate system, based on solutions to their equations of motion.

Findings. Experiments conducted with the developed simulation model made it possible to track the temporal distribution of forces during impact interactions of spherical bodies with a plane, as well as their kinematics – specifically, the duration of contact and the changes in linear and angular velocities at the moment of impact. The computational experiment was carried out using a model of pellet interactions with technological surfaces.

Originality. Unlike the existing models, the newly developed algorithms and simulation model enable continuous monitoring of all the primary kinematic and dynamic parameters throughout the impact event, identifying both the maximum force and impact duration as well as yield an approximation function describing how impact force varies over time when spherical bodies collide with technological surfaces. The proposed algorithms also allow for the simultaneous simulation of multiple-body interactions, such as those occurring in a flow.

Practical value. The obtained results were validated for the interaction of ore pellets with technological surfaces, making it possible to determine a safe approach velocity – one that prevents pellet destruction – when contacting working surfaces. This substantially reduces pellet fragmentation during transportation and loading operations, ultimately improving the quality of ferroalloy smelting from these pellets.

Keywords: *algebraic-logical functions, computational experiment, spheres, discrete element method (DEM), iron ore pellets*

Introduction. Modern geopolitical realities in Ukraine, shaped by the architecture of Ukrainian-American relations, may lead to a new developmental trend in the country's mining and metallurgical industry [1, 2] namely, the extraction of rare-earth metals (REMs). As is well known, against the backdrop of increasing global interest in Ukrainian resources, U.S. President Donald Trump proposed examining the possibility of trading American assistance for access to Ukraine's rare-earth deposits. The use of REMs is critical for the production of cutting-edge high-tech goods, such as smartphones, electric vehicles, and advanced military equipment. In addition, rare-earth elements find extensive application in aerospace and energy industries.

Experts maintain the idea that Ukraine, with its historically significant expertise in mineral extraction, should not only mine these metals but also process them into end products. Naturally, this would require substantial investments, given that REMs are typically obtained through large-scale open-pit mining, a process that consumes vast amounts of energy and can disrupt local ecosystems. Another constraint lies in the fact that all of Ukraine's known deposits are located beneath agricultural lands. Considering the defence implications of jointly developing these resources on Ukrainian territory alongside the United States, it is assumed that the government will find ways to overcome this limitation. A third major challenge pertains to the enrichment processes involved in rare-earth metal production.

To study numerous technological processes of interaction between working bodies and the working environment, the latter is described as a continuous medium and studied by appropriate analytical and numerical methods. In particular, the finite element method is widely used to study continuous media [3, 4]. However, for the study of a system of solid particles (objects), models for continuous media do not allow evaluating the dynamics of individual particles and the processes of their interaction with each other. To study such systems, the discrete element method (DEM), described in detail in [5], has become widespread. In particular, the DEM method has been successfully used to model the transportation system of screw conveyors [6] and the loading of a bunker with grain material [7]. A comparison of DEM modelling with the results of laboratory experiments is presented in [8]. The method has proven itself to be a good tool for modelling the flow of individual particles and determining the distribution of velocities and forces. However, the use of this method requires large computing resources, and the corresponding software is not available for many consumers, in particular in Ukraine, due to its high cost [9]. In addition, its use is complicated when detailing and establishing the distribution of kinematic and dynamic parameters of the movement of individual elements under conditions of simultaneous impact interaction, since the time of such interaction is often thousands of fractions of seconds [10, 11].

In order to clarify the nature of contact interaction in the flow, algorithms for describing objects in a discrete environment by algebraic-logical functions were developed, where the distances between them during approach and contact interaction are determined by the level of the fields of these functions [12], which significantly reduces the procedure for determining the positioning of objects. Such a formalized description also made it possible to simplify the procedure for finding contact zones [13] and the kinematics of objects during rigid convergence [14]. During the movement and collision of particles in the flow and in their contact with the working bodies, there is an impact interaction of objects, which at low speeds and, accordingly, elastic deformations is reduced to the Hertz formula [15, 16]. However, the solution of the contact problem does not allow describing changes in the force of interaction of objects and their kinematics as functions of time. Such regularities are studied by numerical methods by implementing appropriate simulation models, in particular those built using the above approaches [17]. The use of such simulation models makes it possible to implement a computational experiment to establish patterns of change over time of the kinematic and force parameters of impact interaction, which are not derived analytically, and whose experimental determination is complicated by the extremely short time of contact interaction. Paper [18] presents the results of a computational experiment to determine the kinematic and dynamic parameters of the impact interaction of apples with each other and with working surfaces. However, the materials of known studies do not provide analytical or approximation dependencies for determining the force and time of impact contact interaction, which are of significant practical interest.

The aim of the research is to establish the regularities of the time distribution of the kinetic and dynamic characteristics of the contact interaction of objects based on the results of the computational experiment with the construction of appropriate approximation dependencies, as well as to derive dependencies for determining the time and maximum force of contact interaction of objects during impact.

The computational experiment was carried out to study the interaction of spherical objects (particles) with each other and with a flat surface. For this purpose, a simulation model of the objects and the interaction process was used, in which the movement of each individual particle, its shape, and interaction parameters are determined on a computer in full compliance with the real process. In this case, the objects of interaction were written in the form of algebraic logic functions (R-functions), which form a scalar field of unit gradient around the object and have the following appearance, $f_i(x, y, z, t) = 0$ [18].

Methodology and method computational experiment. For an object that is bounded by a spherical surface, the unit gradient function in its own coordinate system has the form of a central (spherical) field function [12, 18].

$$f_i(x,y,z) = \sqrt{x^2 + y^2 + z^2} - r_i, \tag{1}$$

r_i is the radius; i – a sphere identifier.

If $f_i(x, y, z) = 0$, then equation (1) describes the set of points $E_i(x, y, z)$, lying on the sphere (surface of the ball), at $f_i(x, y, z, t) < 0$, respectively, the set of points $B_i(x, y, z)$, lying in the body of the bullet at a depth of f_i from the surface. Points $A_i(z, y, z)$, for whom $f_i(x, y, z, t) > 0$, will lie outside the sphere at a distance f_i to its surface.

For a function of a unit gradient, the normal vector to the surface of the object is equal to the gradient vector of the function, that is $|\mathrm{grad}\, f| = 1$. In particular, for the function (1)

$$\begin{aligned}\bar{\mathbf{n}} = \mathbf{grad}\, f_i(x,y,z) = \\ = \frac{x \cdot \bar{\mathbf{i}}}{\sqrt{x^2 + y^2 + z^2}} + \frac{y \cdot \bar{\mathbf{j}}}{\sqrt{x^2 + y^2 + z^2}} + \frac{z \cdot \bar{\mathbf{k}}}{\sqrt{x^2 + y^2 + z^2}}.\end{aligned} \tag{2}$$

For a sphere (ball), the centre of gravity O_i which in the time of $t = 0$ was located in t. $O_{i0}(x_{i0}, y_{i0}, z_{i0})$ and moving at a speed of $\bar{\mathrm{v}}_{\mathrm{i}} = v_{ix}\bar{\mathbf{i}} + v_{iy}\bar{\mathbf{j}} + v_{iz}\bar{\mathbf{k}}$, dependence (1) will take the form

$$\begin{aligned} f_i(x,y,z,t) = \\ = \sqrt{(x - x_{i0} - v_{ix}t)^2 + (y - y_{i0} - v_{iy}t)^2 + (z - z_{\mathrm{i}0} - v_{iz}t)^2} - \\ - r_з = 0. \end{aligned} \tag{3}$$

The functions of a unit gradient can be used to describe arbitrary working surfaces formed by bodies of revolution or flat surfaces, including moving ones, and their combinations. Thus, a flat surface moving with a speed of $\vec{v}_j$ provided that the direction of the normal vector to the surface is unchanged, will be described by the dependence

$$\begin{aligned} f_i(x, y, z, t) = \alpha_{nx}(x - x_{j0} - v_{jx}t) + \alpha_{ny}(y - y_{j0} - v_{jy}t) + \\ + a_{nz}(z - z_{j0} - v_{jz}t) = 0, \end{aligned} \tag{4}$$

where α_{nx}, α_{nx} and α_{nz} are guide cosines of the angles of inclination of the plane surface normal to the corresponding coordinate axes; x_{j0}, y_{i0}, z_{j0} – coordinates of the base point of the surface at a given time $t = 0$; v_{jx}, v_{jy}, v_{jz} – components of the speed of movement of the working surface $\vec{v}_j$.

Complex objects are described by algebraic and logical functions that are composed of functions that describe individual surface elements.

The running distance from the surface of an arbitrary object, which is described by the unit gradient function $f_i(x, y, z, t) = 0$, to an arbitrary moving point, such as the centre of gravity $O_k[x_k(t), y_k(t), z_k(t)]$ of another object will be defined as

$$l_{ik} = f_i[x_k(t), y_k(t), z_k(t), t]. \tag{5}$$

The rate of convergence of objects of interaction with each other, in particular in a flow, is defined as $v_{ik}(t) = -dl_{ik}/dt$.

Consider the interaction of two objects. In the impact theory, the assumption of instantaneous impact interaction of objects is often used, where the main parameters are determined from the law of conservation of motion. This assumption leads to a significant simplification of the computational model of interaction and cannot be used to describe the simultaneous interaction of several objects, their movement in a flow, etc. where the assumption of instantaneous contact loses its meaning.

One of the most acceptable ways to solve the problems of interaction of several objects with low velocities of approach is to build a model of their elastic interaction based on the solution of the Hertz formula, in which the interaction force is equal.

$$P_{ij} = E_{ij}^* u_{ij}^{3/2} \Big/ \sqrt{K_i + K_j}, \tag{6}$$

where u_{ij} is the value of rigid convergence in the Hertzian contact problem; K_i, K_j – the curvature of the objects surfaces at the point of contact (for curved surfaces with a minus sign); E_{ij}^* – the composite Young's modulus, which takes into account the elastic properties of the conjugation of the contact bodies,

$$E_{ij}^* = \frac{4}{3}\left(\frac{1-\nu_i^2}{E_i} + \frac{1-\nu_j^2}{E_j}\right)^{-1}, \tag{7}$$

where ν_i, ν_j and E_i, E_j are, respectively, the Poisson's ratios and Young's moduli of the materials of the objects of interaction.

The maximum contact stresses in the center of the contact are determined by the following relationship

$$\sigma_{ij\max} = \frac{1}{\pi}\sqrt[3]{6P_{ij\max}E_{ij}^*(K_i + K_j)^2}. \tag{8}$$

For the case of interaction of bodies other than balls, the contact area will not be circular and for the contact surface is determined by the reduced curvature, which, as a first approximation, is assumed to be equal to the average surface curvature $K_i = H_i$.

The amount of rigid convergence u_{ij} was defined in the model as the depth of penetration of one geometric object into another.

For surfaces of a unit gradient

$$u_{ij} = \Delta h_i + \Delta h_j = -[f_{iE}(x_E, y_E, z_E, t) + f_{jE}(x_E, y_E, z_E, t)], \tag{9}$$

where Δh_i and Δh_j depth of penetration of each object relative to the calculated point of contact (convergence) $E(x_E, y_E, z_E)$.

With simultaneous interaction n objects in contact points other than forces P_{ij} from the normal approach of objects, tangential forces arise F_{ij} from their mutual slippage at the point of contact $E(x_E, y_E, z_E)$ [18].

$$F_{ij} = \frac{\mu \Delta \bar{v}_{eij}^{\wedge}}{|\Delta \bar{v}_{eij}^{\wedge}|} P_{ij}, \tag{10}$$

where $\Delta \bar{v}_{eij}^{\wedge}$ is the vector of relative tangential velocity of the surfaces of the objects of interaction, which depends on their linear and rotational movements; μ – sliding friction coefficient.

In general, a spherical object has 6 degrees of freedom, where possible superimposed bindings are replaced by corresponding reactions from other objects. Since slippage is possible at the point of contact, the systems of equations for linear displacements were written in a common (inertial) coordinate system $Oxyz$, and for rotational movements – in its own coordinate system $O_i^{\wedge} x_i^{\wedge} y_i^{\wedge} z_i^{\wedge}$ i – identifier, tied to its principal axes of symmetry [18].

$$\sum_{i=1}^{n} P_{ij}\left[grad(f_i) - \frac{\mu \Delta \bar{v}_{eij}}{|\Delta \bar{v}_{eij}|}\right] - m_i \bar{a}_i + m_i \bar{g} = 0; \tag{11}$$

$$\sum_{i=1}^{n} P_{ij}\left\{(\bar{r}_{ij}^{\wedge} + \bar{\delta}_{ij}^{\wedge}) \times \left[grad(f_i^{\wedge}) - \frac{\mu \Delta \bar{v}_{eij}^{\wedge}}{|\Delta \bar{v}_{eij}^{\wedge}|}\right]\right\} - \bar{L}_{0i}^{(e)} = 0, \tag{12}$$

where $\bar{a}_i$ is a linear acceleration vector of object of mass m_i; $\bar{P}_{ij}$ and $\bar{P}_{ij}^{\wedge}$ are the vectors of normal forces of elastic interaction according to the Hertz model, given, respectively, in the general and proper coordinate systems; $\bar{r}_{ij}$ and $\bar{r}_{ij}^{\wedge}$ – corresponding radius vectors ij –zones in coordinate systems $Oxyz$ and $O_i^{\wedge} x_i^{\wedge} y_i^{\wedge} z_i^{\wedge}$; $\bar{\delta}_{ij}^{\wedge}$ – tangential displacement of the contact area from the force $\bar{F}_{ij}^{\wedge}$, $\bar{\delta}_{ij}^{\wedge} = \bar{F}_{ij}\nu_i \big/ (4a_{ij}G_i)$; a_{ij} – calculated radius of the contact area, $a_{ij} = \sqrt[3]{3P_{ij} \big/ [4E_{ij}^*(K_i + K_j)]}$; G_i – shear module, $G_i = E_i/[2(1+\nu_i)]$; $\bar{L}_{oi}^{(e)}$ – vector sum of force moments $\bar{L}_{0i}^{(e)} = \frac{d\bar{K}_{0i}^{\wedge}}{dt} + ([\bar{\omega}_0^{\wedge} \times \bar{K}_{0i}^{\wedge}])$; $\bar{K}_{0i}^{\wedge}$ – momentum.

For a spherical object with central symmetry $\bar{L}_{oi}^{(e)} = I \cdot \left(\frac{d\omega_x}{dt} \cdot \bar{i} + \frac{d\omega_y}{dt} \cdot \bar{j} + \frac{d\omega_z}{dt} \cdot \bar{k}\right)$, where I is the axial moment of inertia relative to an arbitrary axis; ω_x, ω_y and ω_z – components of the angular velocity of an object relative to its main axes of inertia.

Connection between a common coordinate system $Oxyz$, in which the equation (11), and its own coordinate system $O_i^{\wedge} x_i^{\wedge} y_i^{\wedge} z_i^{\wedge}$ i – object in which the equation is written (12), for every next step Δt the calculations were performed in a homogeneous coordinate system using a modified transformation matrix for elementary linear and angular displacements.

$$M(R) = \Pi(\alpha, l) M(R^{\wedge}) = \\ = \begin{vmatrix} 1 & -\omega_z \Delta t & \omega_y \Delta t & v_x \Delta t \\ \omega_z \Delta t & 1 & -\omega_x \Delta t & v_y \Delta t \\ -\omega_y \Delta t & \omega_x \Delta t & 1 & v_z d\Delta t \\ 0 & 0 & 0 & 1 \end{vmatrix} M(R^{\wedge}), \tag{13}$$

where $M(R)$ is a matrix of coordinates of arbitrary points of an object, including contact zones, in a common coordinate system, $M(R) = (x_E, y_E, z_E, 1)^{-1}$; $M(R^\wedge)$ — the coordinate matrix of the same points in the object's own coordinate system, $M(R^\wedge) = (x_{3E}^\wedge, y_{3E}^\wedge, z_{3E}^\wedge, 1)^{-1}$.

The algorithm for implementing the simulation model involves the following steps.

1. Formation of surfaces of interaction objects by algebraic dependencies.
2. Initial placement of the interaction objects in space, giving them initial kinematic parameters $\overline{v}_0(t)$ and $\overline{\omega}_0(t)$.
3. Bringing the surfaces of objects closer together until contact occurs and selecting a set of contact zones.
4. Determining the forces and moments acting on each of the objects from the other at the initial moment of time t.
5. Definition of linear $\overline{a}(t)$ and angular $\overline{\varepsilon}(t)$ accelerations of each of the objects due to the forceful influence of the others at a given time t.
6. Defining new values for linear $\overline{v}(t+\Delta t) = \overline{v}_0(t) + \overline{a}(t)\Delta t$ and angular $\overline{\omega}(t+\Delta t) = \overline{\omega}_0(t) + \overline{\varepsilon}(t)\Delta t$ velocities at the next moment in time.
7. Providing movements to each of the objects of interaction and fixing the location of objects for a period of time $t + \Delta t$.
8. Formation of new contact zones and determination of force impacts on the object in them.
9. Repeating the calculation cycles until the planned result is achieved.

One of the things that complicates the model based on the implementation of the Hertzian contact problem is that the relationship between interaction forces and displacements is static and does not depend on the time factor. However, the realization of the simultaneous interaction of bodies with each other, for example, in a flow, involves short-term contact (impact) interaction, including simultaneous, in which the time factor is decisive. To determine the change in the forces of contact interaction between two objects and other factors over time, and to establish the contact time based on the developed model, a computational experiment was performed. By analogy with [18], the impact interaction of an iron ore pellets, which was modelled as a ball, with a flat, rigid surface was studied. Such a problem is of considerable practical value in the technological processes of apple processing and storage.

Experiment and Results. To establish the kinetic and force parameters of the contact interaction of objects, the impact interaction of an apple with diameter $D = 0.06$–0.1, which hit a hard flat surface with a certain initial speed, which is determined by the technological process, was modeled by the method of a computational experiment.

The rheological properties of iron ore pellets, depending on their varieties, vary in a fairly wide range and were taken according to studies [19, 20] and reference data. The contact interaction was modelled by dropping a spherical model of an iron ore pellets onto a horizontal and inclined rigid plane with the characteristics of steel used to manufacture technological equipment. During the computational experiment, the coordinates of the object, its linear and angular velocities, the initial values of which can be set arbitrarily, and the distribution of the contact force were recorded.

To implement the computational experiment, we used the existing software, Fig. 1. In particular, Fig. 2 shows the law of change of the contact force for an iron ore pellets with a diameter of $D = 0.015$ mm, the shear modulus of which $G = 60$ MPa, and the Poisson's ratio $\mu = 0.22$, falling under the action of the earth's gravity on a rigid horizontal plane (steel $G = 8.1 \cdot 10^{10}$ Pa, $\nu = 0.28$) for different values of the initial normal component of the velocity of approach of an iron ore pellets to a flat surface $v = [1; 2; 3; 4; 5; 6; 7; 8]$. The characteristics of the coil were taken as averaged according to [19–23].

Fig. 3 shows similar results of the interaction of an iron ore pellets with the same parameters with a similar flat surface placed at an angle $\gamma = 45°$.

It is shown in [18] that in the impact interaction of bodies that have an initial angular velocity of rotation or a transverse component of the linear velocity before approaching, the surfaces will have a mutual tangential displacement in the contact zone due to slippage. Similarly, when an apple strikes an angled platform, it reflects in the direction transverse to the initial motion. In particular, when iron ore pellets hits a platform set at an angle vertically $\gamma = 45°$ to the plane xOy and parallel to the axis Oy, it will acquire a linear velocity component v_x along the axis Ox, and angular velocity ω_y relative to the axis Oy. The growth of these velocities will occur during the entire time of contact interaction of a spherical object (in this case, an iron ore pellets) according to the law, which can be approximately described by the logistic function given in Figs. 4 and 5. The parameters of the iron ore pellets and the rheological properties of the interaction objects are fully consistent with the data of the computational experiment in Fig. 3.

Similar studies were conducted for other parameters of the objects of interaction. In addition, the results described in other sources were analyzed. Processing of the obtained results showed that the distribution of the contact interaction force in time is characterized by a bell-shaped curve close to the Gaussian function, which at the edges, i. e., at the initial moment of contact $t = 0$ and at the end of the day, $t = t_k$, takes zero values, $P = P_{ij} = 0$. Here t_k is time of contact interaction. In addition, the studied distribution is characterized by a slight

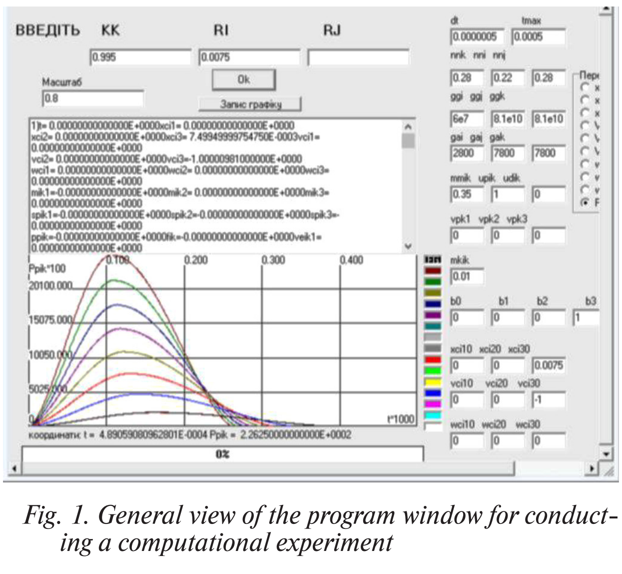

Fig. 1. General view of the program window for conducting a computational experiment

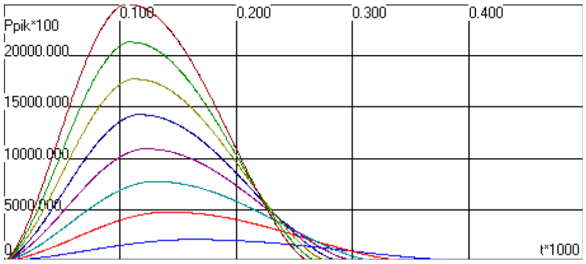

Fig. 2. Time variation of the impact force P iron ore pellets of a diameter D = 0.015 *m to a horizontal flat steel surface at approach speeds* (*m/s*):

1 – $v_1 = 1$; *2* – $v_2 = 2$; *3* – $v_3 = 3$; *4* – $v_4 = 4$; *5* – $v_5 = 5$; *6* – $v_6 = 6$; *7* – $v_7 = 7$; *8* – $v_8 = 8$

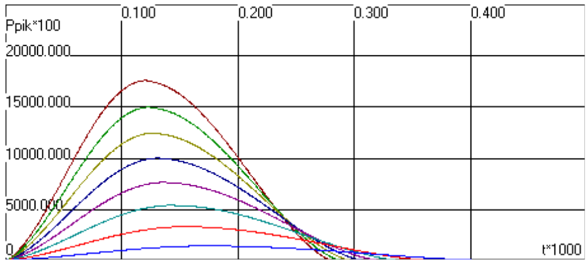

Fig. 3. Time variation of the impact force P iron ore pellets of a diameter D = 0.015 *m to an inclined flat steel surface installed at an angle* $\gamma = 45°$ *at convergence speeds* (*m/s*):

1 – $v_1 = 1$; *2* – $v_2 = 2$; *3* – $v_3 = 3$; *4* – $v_4 = 4$; *5* – $v_5 = 5$; *6* – $v_6 = 6$; *7* – $v_7 = 7$; *8* – $v_8 = 8$

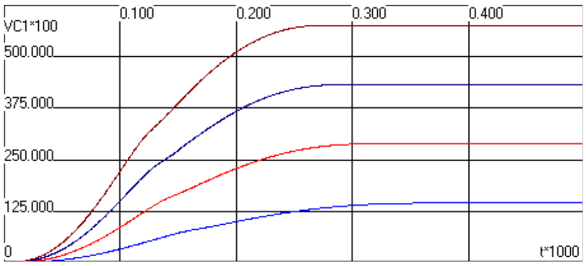

Fig. 4. Changing the horizontal speed of an iron ore pellets of a diameter D = 0.015 *m to an inclined flat steel surface at an angle* $\gamma = 45°$ *to the horizon at the initial speed of approach* (*m/s*):

1 – $v_1 = 2$; *2* – $v_2 = 4$; *3* – $v_1 = 6$; *4* – $v_4 = 8$

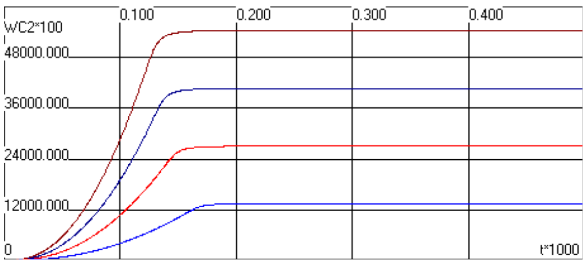

Fig. 5. Changing the angular velocity of an iron ore pellets of a diameter D = 0.015 *m to a flat steel surface installed at an angle* $\gamma = 45°$ *to the horizon at the initial approach speeds* (*m/s*):

1 – $v_1 = 2$; *2* – $v_2 = 4$; *3* – $v_1 = 6$; *4* – $v_4 = 8$

asymmetry. These factors served as the basis for finding the best distribution curve to approximate the constructed dependencies of the contact force change on time. Let us introduce an additional dimensionless time variable, $\tau = t/t_k$, the scope of which $0 \le \tau \le 1$. Then the distribution of the interaction force in the contact time interval can be approximated by the dependence

$$P_{ij}(\tau) = P_{ij\max} t_k \left[\left(\frac{\tau}{\tau_{\max}} \right)^{\tau_{\max}} \left(\frac{1-\tau}{1-\tau_{\max}} \right)^{(1-\tau_{\max})} \right]^{\varepsilon} = P_{ij\max} t_k f(\tau), \quad (14)$$

where $P_{ij\max}$ is maximum force of contact interaction; ε – parameter of the curve shape, for elastic impact $\varepsilon = 4...5$; $\tau_{\max}$ – asymmetry parameter, $\tau_{\max} = t_{P\max}/t_k$, corresponds to the value τ, at which the force of contact interaction reaches a maximum; $t_{P\max}$ – the time from the beginning of the contact during which the contact force reaches its maximum, $P_{ij} = P_{ij\max}$; $f(\tau)$ – a dimensionless function of contact force distribution during impact interaction.

The form of dependence (14) fully corresponds to both the distribution curves shown in Figs. 2 and 3 and the results obtained for other characteristics of the objects of interaction. In particular, Fig. 6 shows the law of change $P = P_{ij}$ of the contact interaction force for a ball with a diameter $D = 0.015$ m, the Poisson's ratio $\mu = 0.22$ the material of which is the shear modulus of which is $G = 60$ MPa, which falls with a speed under the action of gravity onto a rigid horizontal plane (material steel $G = 8.1 \cdot 10^{10}$ Pa, $\mu = 0.28$) for different values of the initial normal component of the velocity of the ball approaching a flat surface $v = [1; 2; 3; 4; 5; 6; 7; 8]$. Accordingly, Fig. 7 shows the law of change of the contact interaction force for balls of different diameters with the same physical characteristics of the material.

As follows from Fig. 7, the size of the ball has the greatest influence on the impact force, and therefore on the contact stresses that can lead to the destruction of the ball. In this case, the main factor that can be controlled by changing the force effects on the balls and the speed of its collision with technological surfaces, Figs. 2, 3. Fig. 8 shows the dependence of the impact force $P(N)$ of balls with a diameter $D = 0.015$ m on a horizontal flat and inclined $\gamma = 45°$ steel surface according to the results of implementing the collision model shown in Figs. 2,3.

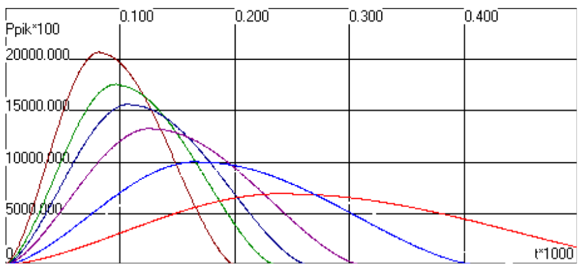

Fig. 6. Time variation of the impact force P of iron ore pellets of diameter D = 0.015 *m to a horizontal flat steel surface for different values of its shear modulus G MPa:*

1 – $G = 10$; *2* – $G = 25$; *3* – $G = 50$; *4* – $G = 75$; *5* – $G = 100$; *6* – $G = 150$

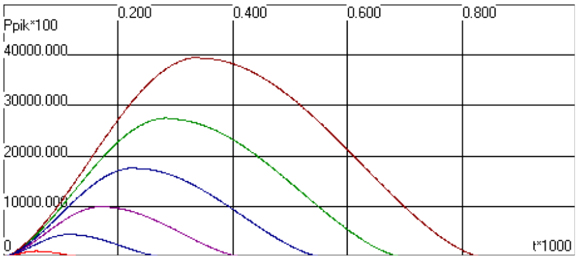

Fig. 7. Time variation of the impact force iron ore pellets P of different diameters D (m) against a horizontal flat steel surface:

1 – $D = 0.003$; *2* – $D = 0.005$; *3* – $D = 0.010$; *4* – $D = 0.015$; *5* – $D = 0.020$; *6* – $D = 0.025$

Fig. 9 shows a graph of the dimensionless distribution function of the impact forces $f(\tau)_k$, according to dependence (14) at different values ε – asymmetry parameters τ_{max} and the shape of the distribution curve ε, whose values are adjusted according to the results of the calculated experiment.

The analysis of the results of the computational experiment showed that for the case of contact interaction of iron ore pellets with a rigid surface, the asymmetry coefficient is insignificant and varies from $\tau_{max} = 0.45$, for the velocities of mutual approach of objects near $v_{nij} = 1$ m/s, to $\tau_{max} = 0.48$–0.49 for $v_{nij} \le 0.4$ m/s. To simplify the model, the change in force $P_{ij\max}$ and time t should be approximated by a symmetric curve ($\tau_{max} = 0.5$). Then the dependency (14) is significantly simplified

$$P_{ij}(\tau) = 4P_{ij\max}t_k[t(1-\tau)]^{\varepsilon/2}. \quad (15)$$

The condition for the conservation of momentum at impact, taking into account (14), is written in the following form

$$m_i v_{nij}(1+e) = \int_0^{t_k} P_{ij}dt = k_P P_{ij\max} t_k, \quad (16)$$

where e is the coefficient of restitution during the impact; k_P is a dimensionless specific impulse that takes into account the shape of the curve (14). The methodology for determining the impact recovery factor is given in [19].

Here k_P is the dimensionless specific impulse of motion, which takes into account the shape of the curve (14) and, according to (16),

$$k_P = \int_0^1 f(\tau)d\tau = \int_0^1 \left[\left(\frac{\tau}{\tau_{\max}}\right)^{\tau_{\max}} \left(\frac{1-\tau}{1-\tau_{\max}}\right)^{(1-\tau_{\max})}\right]^{\varepsilon} d\tau. \quad (17)$$

For a symmetric curve $\tau_{max} = 0.5$ with the form parameter $\varepsilon = 4$ the specific impulse will be $k_P = 0.533$ (for $\varepsilon = 5$ respectively $k_P = 0.49$).

The analysis of the results obtained from the computational experiment, Figs. 2, 3, Fig. 6, showed that the values of the coefficients k_P, which corresponds to the momentum ratio $p = m_i v_{nij}(1 + e)$, that is, the area under the curve $P_{ij}(\tau)$, to the square $S_P = P_{ij\max}t_k$ varies from $k_P = 0.45$, for the velocities of mutual approach of objects near $v_{nij} = 1$ m/s, to $k_P = 0.49$ for lower speeds $v_{nij} \le 0.4$ m/s. The greatest correspondence of the approximation dependence of the function of change in the contact force of interaction as a function of time is observed for the parameter of the curve shape $\varepsilon = 4$–5. At the same time, the value of the specific impulse calculated by dependence (17) and by the ratio of areas on the graph obtained from the results of the computational experiment does not exceed 1 %. Based on the results of the computational experiment, it can also be concluded that such a model parameter as the ratio of the maximum contact force $P_{ij\max}$ to the time of contact interaction t_k i.e. $\chi_P = P_{ij\max}/t_k$, which is a parameter of the curve's peak and depends on the physical characteristics of the interaction objects and their convergence rate and, practically, does not depend on their size, which is confirmed by the data presented in Fig. 7 and can be used as a characteristic of specific interaction conditions, which can be easily calculated from the results of a computational experiment. Influence of parameters ε and τ_{max} is minimal and can be taken as model constants. Taking into account the ratio $\chi_P = P_{ij\max}/t_k$ dependence (18) will take the form $m_i v_{nij}(1+e) = k_P P^2_{nij\max} / \chi_P$.

Accordingly, the maximum contact force will be defined as.

$$P_{nij\max} = \sqrt{\chi_p m_i v_{nij}(1+e)/k_P}. \quad (18)$$

The time of contact interaction is determined by the dependence

$$t_k = \sqrt{m_i v_{nij}(1+e)/(k_P \chi_p)}. \quad (19)$$

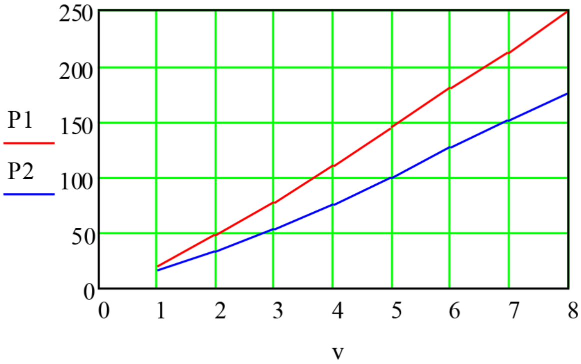

Fig. 8. Dependence of the impact force P(N) of iron ore pellets of a diameter D = 0.015 m on a horizontal flat (1) and inclined at an angle (2) steel surface on the speed of approach v (m/s)

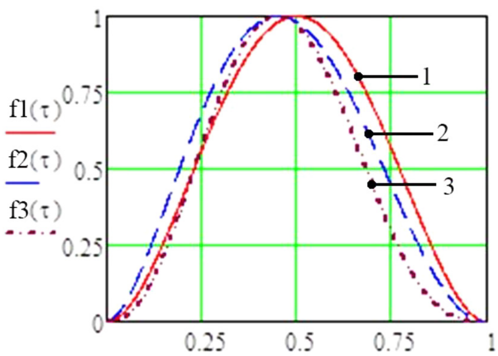

Fig. 9. An approximation dependence of the distribution is given f(τ) for:

1 – $\varepsilon = 4$; $\tau_{max} = 0.5$; *2* – $\varepsilon = 4$; $\tau_{max} = 0.45$; *3* – $\varepsilon = 6$; $\tau_{max} = 0.45$

The studies have confirmed that the maximum contact forces and maximum normal contact stresses, which are determined by dependence (8), are characteristic of the contact interaction of large objects, i.e., large iron ore pellets. Therefore, from a practical point of view, it is important to know the value of the curve peak parameter for technological calculations ξ_P for certain collision speeds of iron ore pellets technological surfaces.

So for iron ore pellets of a diameter $D = 0.015$ m, at collision speeds $v = 1$ m/s, the parameter χ_P will take the value $\chi_P = 5.2 \cdot 10^4$ N/s, at speeds $v = 5$ m/s, respectively $\chi_P = 5.1 \cdot 10^5$ N/s, and at $v = 8$ m/s – $\chi_P = 9.7 \cdot 10^5$ N/s.

Dependences (18 and 19), based on the known model parameters, allow us to calculate such important parameters as the contact interaction time and the maximum force and stress (pressure) in the contact zone and can be used in the relevant technological calculations. The results obtained from the computational experiment are generally consistent with those obtained by other researchers.

Conclusion. The paper presents a computational experiment to establish the regularities of contact interaction of elastic objects in particular, iron ore pellets of the metallurgical industry with technological surfaces and determine the distribution of contact forces in time. For this purpose, a simulation model based on algorithms for finding the location of objects, determining contact zones, and calculating the distribution of contact forces and kinematics of objects during interaction is used. The developed algorithms and model allow realizing simultaneous interaction of bodies, including in a flow.

In order to reduce the computational time, an analytical dependence is proposed to approximate the distribution of contact forces during the interaction of objects, which combines the time and maximum contact force of interaction and the parameters of the shape and asymmetry of the distribution curve. Based on the results of the computational experiment, the parameters of the distribution of the approximation curve are established and dependencies for determining the time and maximum contact pressure forces are proposed.

The implementation of the computational experiment was carried out using iron ore pellet model, since such studies are of practical interest, which makes further verification of the model in real experimental conditions relevant.

It was found that the approximation model adequately reproduces the time distribution of contact interaction forces. With an appropriate choice of the curve shape parameter ε, the impulse values calculated from the computational experiment and the approximation curve differ by no more than 1 %.

It is shown that the contact interaction time decreases with increasing speed of convergence of objects, and for a sufficiently large range of speeds, the change in contact time is insignificant, which made it possible to propose dependencies for determining the time and maximum force of contact interaction, which is of practical interest for technological calculations.

According to the results of the experiment, it was found that in an indirect collision of bodies, the change in their linear and angular velocities during the contact interaction occurs according to a law that can be described by a logistic curve.

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Реалізація обчислювального експерименту ударної взаємодії сферичних тіл

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Мета. Встановлення закономірностей зміни у часі кінетичних і динамічних параметрів ударної контактної взаємодії об'єктів за результатами реалізації обчислюваного експерименту з побудовою відповідних апроксимаційних залежностей для їх використання в розробці технологічних процесів завантаження тіл сферичної форми, зокрема котунів.

Методика. Для встановлення законів зміни у часі кінематичних і динамічних параметрів контактної взаємодії об'єктів і побудови відповідної моделі обчислюваного експерименту був використаний метод імітаційного моделювання, що базується на розв'язку контактної задачі Герца й моделях зближення об'єктів у однорідній системі координат за результатами розв'язку диференціальних рівнянь їх руху.

Результати. Проведені експерименти на розробленій імітаційній моделі дозволили встановити розподіл у часі сил при ударній взаємодії тіл сферичної форми із площиною та їх кінематику із визначенням часу контакту та зміни лінійних і кутових швидкостей тіл у момент удару. Реалізація обчислюваного експерименту проводилася на моделі взаємодії котунів із технологічними поверхнями.

Наукова новизна. Розроблені алгоритми й імітаційна модель, на відміну від існуючих, дозволяє прослідкувати зміну у часі всіх основних кінематичних і динамічних параметрів під час ударної взаємодії, встановити максимальну силу й час удару та вивести апроксимаційну залежність зміни у часі сили ударної взаємодії тіл сферичної форми з технологічними поверхнями. Розроблені алгоритми також дозволяють імітувати одночасну взаємодію багатьох тіл, зокрема в потоці.

Практична значимість. Отримані результати апробовані для взаємодії котунів рудного матеріалу із технологічними поверхнями й дозволяють встановити безпечну, із умови їх не руйнування, швидкість зближення з робочими поверхнями. Це суттєво зменшить подрібнення котунів рудного матеріалу під час транспортних і завантажувальних операцій і підвищить якість виплавки феросплавів із котунів.

Ключові слова: *алгебро-логічні функції, обчислювальний експеримент, кулі, метод дискретних елементів, залізорудні котуни*

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