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## CHARACTERISTICS OF ELASTICITY, FREQUENCY, AND STABILITY OF PLATE CONNECTING ASSEMBLIES FOR VIBRATING MACHINES

**Purpose.** To formalize the design of the elastic element of the connection unit of a multi-axis vibrating machine with a plate, in the form of internal cutouts and autonomous jumpers. To obtain the dependence of the stiffness of the plate elastic element in the direction of transmitted vibration on the modulus of elasticity of the plate material, the cross-sectional area of an individual lintel, its length, width, and thickness. To determine the limiting values of the forcing forces of the machine that satisfy the stability conditions of the jumpers of the plate package according to Euler.

**Methodology.** When creating the configuration of an elastic element in the form of longitudinal bridges of an even number and modeling the stiffness characteristic of the joint, the methods of applied mechanics and the theory of material resistance were used.

**Findings.** The basic results of the work include mathematical and physical modeling of the characteristics of elasticity, stability, and mutual influences in the working and combined directions in relation to the connection points of multi-coordinate vibration machines, which are implemented through flat multi-section plate elements. Taken together, this makes it possible to create energy-efficient vibration machines, to increase the accuracy of the program movement of their working body, which in turn increases: the productivity of mining equipment – in the extraction of minerals, and their reliability – in the case of mechanical tests.

**Originality.** It consists of the fact that for the first time the dependence of the stiffness of the plate elastic element in the direction of transmitted vibration on the modulus of elasticity of the plate material, the cross-sectional area of an individual lintel, its length, width, and thickness were obtained. It is shown that the stiffness of the plate in the conjugate direction is directly proportional to the material's modulus of elasticity, the width of the lintel, its thickness cubed, and is inversely proportional to the length of the lintel in the third degree. It is established that in order to ensure the stability conditions of the lintels of the plate package with respect to the limiting values of the forces of the vibration exciters of the machine, there is their directly proportional dependence on the number of plates in the package, the modulus of elasticity of the material, the width of the lintel and the thickness of the plate in the third degree, and is inversely proportional to the length of the jumper in the square.

**Practical value.** For the first time, a structural mechanical model of an elastic element has been proposed, represented by a plate divided by internal straight slots into external and internal lintels identical in width and length. A real plate connection unit has been created, and its experimental stiffness characteristics depending on the number of plates in the package are given.

**Keywords:** *vibrating machine, plate elastic element, stiffness, stability, operating frequency range*

**Introduction.** The wide scope of application of vibration in technology is due to the possibility of implementing oscillatory motion by the working body of the machine, the amplitude and frequency ranges of which can vary within significant limits. At the same time, the operation of the machine unit in the resonance mode provides significant energy efficiency while reducing the power of control signals. This necessitates research in the field of design and modeling of relevant elements, as well as the development of methods for their calculation. In the theory of vibration reliability, it is relevant to conduct bench mechanical tests of components and assemblies of newly created construction [1], transport [2], aviation [3] and space [4] equipment.

As for construction equipment, these are vibration machines exclusively horizontal  $y(t) = A_y \sin(\omega_y t)$ , vertical  $z(t) = A_z \sin(\omega_z t)$ , or compatible operating principles with switching and configuring operating modes. Therefore, to exclude the transformation of the installation operation schemes, it is necessary to create universal two-coordinate vibration machines, which belong to horizontal-vertical or horizontal-horizontal translational stands. At the same time, such installations belong to special vibration stands, when the trajectories of the control points of the test object in space are flat.

Concerning aviation and space technology, single-axis, two-axis, and three-coordinate installations are usually used, as well as mechanical and climatic test benches, which include vibration, temperature, pressure, and humidity tests at the same time.

Here, the most effective and promising are vibration shock technologies, as well as mechanical tests for shock [5, 6] and spatial [7, 8] multicomponent vibration, when the platform of

the vibration machine gives the test object vibration simultaneously in two or more directions. In directions, for example, with a combination of linear and angular oscillations during the transport vibration test [9].

In this case, the object of study is exposed to a three-component vibration: simultaneously vertical translational and two angular (around the vertical and horizontal axes).

Concerning transport vibration, basic vertical vibrations classically take place  $z(t) = A_z \sin(\omega_z t)$ , as well as angular components  $\varphi(t) = A_\varphi \sin(\omega_\varphi t)$ ,  $\delta(t) = A_\delta \sin(\omega_\delta t)$  around two orthogonal horizontal axes.

Moreover, the frequencies of oscillatory motion in the direction of each coordinate most often correspond to the amplitude-frequency characteristics of the path or the natural frequencies of the nodes of a multidimensional structure, which are subjected to tests for vibration strength or stability of functioning [10].

The use of vibration shock technologies is also effective in mine workings [11, 12].

In this case, the working body of the machine simultaneously performs, for example, the specified axial oscillations  $x(t) = A_x \sin(\omega_x t)$ , as well as torsional ones  $\varphi(t) = A_\varphi \sin(\omega_\varphi t)$  around the central axis.

In this work, an integrated approach and system analysis were used [13] to solve the problem, when the object of research is machines in the form of vibration stands, the platform of which can reproduce shock loads [14], and move separately as in a vertical  $z(t)$ , and horizontal  $y(t)$  directions, as well as simultaneously in the two indicated directions in the plane  $yOz$ .

The type of vibration exciters of the stand, as well as their number, significantly affects the shape and orientation in the space of the trajectory of the working point of the test object [1].

The radius vector of the control point of the platform in a fixed coordinate system under the conditions of a working stand will be as follows

$$\vec{r}(t) = x(t) \cdot \vec{i} + y(t) \cdot \vec{j} + z(t) \cdot \vec{k},$$

where  $\vec{i}, \vec{j}, \vec{k}$  are orthonormal axes of a fixed coordinate system  $Oxyz$ .

From this equation, we obtain the dependence of the trajectory, that is, the hodograph of the vector  $r(t)$ , from the laws of time change of functions  $x(t), y(t), z(t)$ .

Then, if

$$x(t) = Af(t); \quad y(t) = Bf(t); \quad z(t) = Cf(t),$$

the vector of vibration influence will have stable guide cosines

$$\alpha_x = \alpha_y = \alpha_z = 1/(A^2 + B^2 + C^2)^{1/2} = \text{const},$$

and the oscillations are single-axis.

At

$$x(t) = 0; \quad y(t) = Bf_y(t); \quad z(t) = Cf_z(t),$$

the oscillations of the working body will be two-coordinate and belong to the plane  $yOz$ .

At the same time, the trajectories of the points of fixation of the test object on the desktop of the stand can be: a segment of a straight line at an angle  $\varphi = \arctan(k)$ , if  $z(t) = ky(t)$ , for example, when  $z(t) = A_z \sin(\omega_z t)$ ,  $y(t) = A_y \sin(\omega_y t)$  get  $\varphi = \pi/4$ ; circle radius  $R = A$ , at  $z(t) = A \sin(\omega t)$ ,  $y(t) = A \cos(\omega t)$ , and other curvilinear trajectories in the plane  $yOz$ .

Resource and durability [14] units and blocks that have passed bench tests, as well as individual structural elements, depends on the accuracy and reliability of multivariate modeling [15] and the reproduction of the normative mechanical load program at the control point of the platform. The latter, in turn, depends on the kinematic, dynamic and mechanical characteristics of the connection nodes of the oscillation exciter tables with the vibration stand platform [16]. Here, the kinematic characteristics describe the mutual influence between the coordinates of the reproduced vibration in the static mode of the stand operation, when there are no control signals of the stand. At the same time, the static equations of motion of a multi-component platform of a vibration machine must be unconnected, that is, autonomous, which is achieved structurally at the stage of its creation. By analogy with [17], it is accepted that the resource of vibration-transmitting elements is mainly influenced by dynamic characteristics, which depend on the nonlinear components of the equations describing the oscillations of the platform under conditions of manifestation of the imbalance of inertial forces of parts and assemblies of the test object. In this case, the centers of mass, stiffness, and damping of the mechanical system "platform—test object" do not coincide. This causes the complex movement [1, 10] of individual parts of the assembly due to the appearance of parasitic transfer forces of inertia

$$F_1 = |m(\vec{a}_1 + \vec{\varepsilon}_1 \times \vec{\rho} + \vec{\omega}_1 \times (\vec{\omega}_1 \times \vec{\rho}))|.$$

Here  $|\cdot|$  is vector modulus;  $\vec{a}_1$  — linear acceleration vector of the platform;  $\vec{\omega}_1, \vec{\varepsilon}_1$  — vectors of angular velocity and angular acceleration of the platform;  $m, \vec{\rho}$  — mass and radius vector of the center of mass of the node. Also at the same time Coriolis forces of inertia

$$F_2 = |2m(\vec{\omega}_1 \times \vec{V}_2)|,$$

where  $\vec{V}_2$  is the relative velocity vector of a node.

This confirms the relevance of research on the theory, experiment, and practice of creating vibration transmission units of modern machines for testing objects of spatial structure for multicomponent vibration. In this case, the vector of displacements of the control point of the working body of the machine varies in time both in magnitude and in direction. Therefore, a complex stress state occurs in the material of structural parts,

characterized by tensile deformations — compression, bending, and torsion. It most reliably [18] determines the stress state in the material in operation.

**State of the issue.** Practice shows [19], that the quality of mechanical tests for multicomponent translational vibration directly depends on the parasitic torsional vibrations of the platform associated with the tests. It is also negatively affected by the levels of mutual influences between the movements of the platform in the coupled directions, caused by parasitic static deformations of vibration transmission units.

Structurally, these problems [7, 8] have been solved in electromagnet, rubber-metal, and ground-in connection nodes. Partially — in rod, tape elliptical vibration-transmitting nodes. The basic disadvantages of electromagnetic and lapped units include the limited and non-stationary levels of vibration reproduced. Rubber-metal joints are characterized by a critical drawback, which lies in the contradiction of the longitudinal and transverse stiffness of a multilayer washer. The negative features of rod and elliptical nodes are the multi-link connecting rods with spherical hinges at the ends, placed between the tables of vibrating exciters and the platform, as well as devices for tensioning elliptical tapes. Here, each structural unit has its resonance, which distorts the overall amplitude and frequency response of the stand.

**Purpose of the work.** The work aims to create plate packet nodes for connecting the tables of vibrating exciters with the platform of the vibrating machine, which has a given longitudinal rigidity in the direction of the transmitted vibration. Its value also determines the value of the resonant frequency of the plate package. At the same time, they are characterized by both a linear characteristic of transverse stiffness and the absence of mutual influences between the coupled coordinates in the mode of static deformation of the plate elements of the package.

It is also necessary to develop a method for calculating the design parameters and mechanical characteristics of plate elastic elements of vibration transmitting units, which take into account the normative levels of mechanical loads, the operating frequency ranges of the vibration machine, as well as the parameters of the accuracy of the reproduced vibration in the working directions of the stand.

**The main material.** This work is devoted to the development of the design, theoretical, and experimental studies of vibration-transmitting units made in the form of packages of lamellar elastic elements. A structural model of an elastic element is shown in Fig. 1.

It is represented by a plate divided by internal straight slots into internal lintels 1 and 2 external ones identical in width and length. Internal lintels are structurally implemented using a rectangular cutout 3. Fastening of the plate to the vibration exciter table and the stand platform is carried out in sections 4, 5, respectively. Here  $\vec{F}_z, \vec{F}_y, F_z, F_y$  are single and actual forces in the direction of the axes  $Oz$  and  $Oy$ .

Let us further consider the basic static characteristic of the connection points of a multidimensional vibration machine [1]: the absence of dependence of the coordinates of the platform movements on the movements in the direction of the coordinates.

In this case (Fig. 1), when moving horizontally in the direction of the  $Oy$  axis connected to the platform of section 4 of the elastic element, transverse elastic deformations of bridges 1, 2 occur.

In this case, the upper ends of both jumper 1 will be mixed simultaneously in the vertical downward direction, and the lower ends of jumper 2 will be mixed upwards. The width and length of lintels 1 and 2 are the same, so the generalized vertical of the specified section of the plate at the junction with the platform will be the same in size. As a result, the interaction between the coupled directions of the vibration machine will be excluded in a constructive way at the stage of creating the plate. That is, autonomy between its coordinates is achieved.

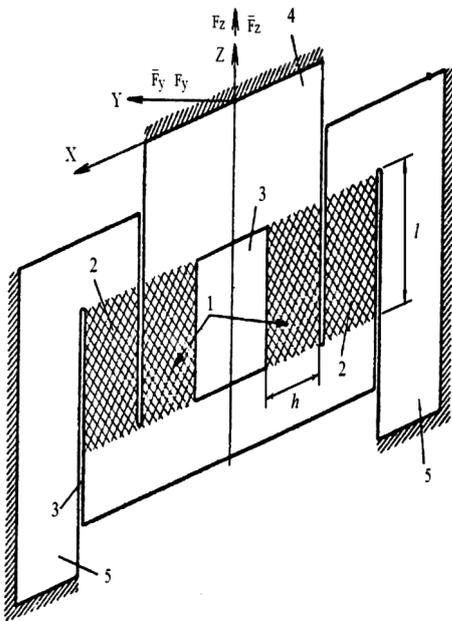


Fig. 1. Appearance of the elastic element

Functionally, at  $F_z \neq 0, F_y = 0$  stiffness  $c_z$  of the element in the axis direction  $Oz$  (working direction of the vibrating stand) is determined by the operation of two jumpers at the same time  $l$  compression plate and two jumpers 2 – tensile (Fig. 1).

Then we get

$$c_z = \frac{\bar{F}_z}{4\Delta l} = \frac{\bar{F}_z}{4\epsilon l} = \frac{\bar{F}_z}{4\left(\frac{\sigma}{E^*}\right)l} = \frac{\bar{F}_z}{4\left(\frac{\bar{F}_z}{4AE^*}\right)l} = \frac{E^*A}{l}, \quad (1)$$

where  $l, \Delta l, \epsilon$  are length, absolute and relative elongation of bridges 1, 2 under longitudinal deformation;  $A$  – cross-sectional area of lintels 1, 2;  $\sigma$  – normal tension-compressive stress in the bridge material;  $E^*$  – modulus of elasticity of the plate material (Fig. 1).

Let us take into account that the area of an individual lintel  $A = hb$ , then we get

$$c_z = E^*h\left(\frac{b}{l}\right), \quad (2)$$

where  $h, b$  are the width of the lintel and the thickness of the plate.

Let us further consider the deformation of the plate in the direction of the axis  $Oy$ , which corresponds to the displacement of section 4 of the plate connected to the platform of the stand, in the direction of the coordinate of the plate conjugated to sections 5, rigidly connected to the table of the vibrating exciter. Each of the lintels is presented in the form of a cantilever beam with length  $l$  and cross-section  $A = hb$ .

Also functionally, at  $F_z = 0$  и  $F_y \neq 0$  stiffness of the element in the axis direction  $Oy$  (combined direction of the vibration stand) is determined by the work of all jumpers 1, 2 of the plate on bending at the same time.

According to the method of forces [1]

$$c_y = \frac{1}{\delta_{11}},$$

where  $\delta_{11}$  is displacement of section 4 of the plate from unit force  $\bar{F}_y = 1$ .

For the elastic element under consideration, the application of the formula approach to the calculation of the Mohr integral results in the expression

$$\begin{aligned} \delta_{11} &= \frac{1}{4E^*I_x} \int_0^l \bar{M}\bar{M}d\eta = \frac{1}{4E^*I_x} \int_0^l (\eta-1)(\eta-1)d\eta = \\ &= \frac{(\eta-1)^3}{12E^*I_x} \Big|_0^l = \frac{l^3}{12E^*I_x}, \end{aligned}$$

where  $\bar{M} = (\eta-1)$  is a bending moment in the cross-section of the cantilever bar-jumper with the coordinate  $\eta$  from unit force in the axis direction  $Oy$ ;  $I_x$  – a moment of inertia of the bar section. Here, the coefficient 4 in the denominator of the formula takes into account the fact that in the direction of action  $F_y$  all four bridges of the plate are deformed to the right at the same time.

Then we get, given that  $I_x = \frac{hb^3}{12}$  and

$$\delta_{11} = \frac{l^3}{12E^*\left(\frac{hb^3}{12}\right)} = \frac{l^3}{E^*hb^3}, \quad (3)$$

the following formula for determining the stiffness of the element in the axis direction  $Oy$

$$C_y = \frac{1}{\delta_{11}} = \frac{E^*l\left(\frac{b}{l}\right)^3}{E^*lb^3}. \quad (4)$$

The peculiarity of the load of the plate of the connection node at  $F_z > 0, F_y = 0$  is that the force applied to section 4 of the plate causes the compression deformation of the bridges only 2. At  $F_z < 0$  only the jumpers of 1 plate will be compressed. In this case, the design parameters of lintels 1, 2 must meet the stability conditions according to Euler, and according to magnitude of the force  $F_z$  must not exceed the critical value  $[F_{cr}]$ .

For a package of plates, we get

$$F_z < [F_{cr}] = \frac{2N\pi^2 E^* I_x}{l^2} = \frac{N\pi^2 E^* hb^3}{6l^2}. \quad (5)$$

In mechanical tests for vibration reliability, the forcing forces  $F_y$  and  $F_z$  applied to the plate are of a variable nature. Therefore, the structural and mechanical parameters of the lintels must also simultaneously satisfy the criterion of fatigue strength of the plate material

$$\sigma \leq k^{-1}\sigma_{-1}, \quad (6)$$

where  $\sigma$  is equivalent stress in plate material;  $\sigma_{-1}$  – material endurance limit;  $k$  – fatigue strength factor.

Let us further assume that the coercive forces transmitted (Fig. 1) through the packet  $F_z > 0, F_y = 0$ . In this case, both jumpers 2 will work in compression at the same time, and jumpers 1 will stretch. Then the normal stresses in the jumpers will be

$$\sigma = \frac{F_z}{4Nhb}, \quad (7)$$

where  $N$  is the number of plates in the connection node. Here, the coefficient 4 in the denominator of the formula takes into account the fact that in the direction of action of the vertical force  $F_z$ , all four bridges of the plate are deformed at the same time.

Let us take into account further in equation (6) the formula (7). Then we obtain an expression for determining the number of plates  $N$ , in which for the accepted structural ( $h, b$ ) and mechanical ( $\sigma_{-1}$ ) parameters of the jumpers and the connection unit, as well as the standard level of the forcing force  $F^*$  reproduced at the control point of the platform, the criterion of the fatigue strength of the package is formalized in the form

$$N \geq \frac{F^*k}{4hb\sigma_{-1}}. \quad (8)$$

Fig. 2 shows the dependence  $N_f(b)$  (curve 1), which corresponds to the conditions of the fatigue strength criterion of the package, taking into account the specified values of the mechanical and structural parameters of flat elastic elements, as well as the normative value  $F^*$ .

In this case, the following mechanical and geometric parameters of the elastic plates of the connection unit are used:  $F^* = 5,000$  H,  $k = 2$ ,  $\sigma_{-1} = 50$  MPa,  $h = 1.5 \cdot 10^{-2}$  m. The curve corresponds to the dependence

$$N_F = 3.33 \frac{1}{b_1},$$

where  $b_1 = [0.5; 1.0; 1.5]$ .

Here, the value of  $N_F$  of the number of plates in the package is rounded to the nearest integer. That is, in the graphs  $N_F = [3; 4; 7]$  according to the thickness.

The structural and mechanical parameters of packages of flat elastic elements in each working (vertical and horizontal) directions must simultaneously satisfy the criterion of rigidity

$$f = \frac{1}{2\pi} \left( \frac{c}{m} \right)^{\frac{1}{2}} = 0.159 \sqrt{\frac{NE^*hb}{l}} \geq f^*, \quad (9)$$

where  $f, f^*$  are frequencies calculated and specified by the normative document of the vibration stand in the working direction;  $m$  – the mass of the movable system of the vibration stand (platform, test object);  $c$  – the stiffness of the connection assembly in the direction of the transmitted vibration, which consists of  $N$  plates.

By analogy with the analytical-structural method of machine design [20], we determine for formula (9) the ratio that must be satisfied by the number of  $N$  plates in the connection unit, taking into account the criterion of rigidity, that is, the given value of the frequency of the working direction

$$N \geq 39.48 \frac{l}{E^*hb} (f^*)^2. \quad (10)$$

Expression (10) additionally establishes the dependence of the number of plates in the package on mechanical and structural plate parameters: modulus of elasticity  $E^*$  plate material of its thickness  $b$ , as well as the length  $l$  and width  $h$  of the lintels.

Dependency  $N_f(b)$ , which meets the conditions of the criterion of stiffness in the direction of transmitted vibration, is shown by curve 2 in Fig. 2. It is built on the basis of the following mechanical and geometric characteristics of the elastic plates of the joint unit:  $f^* = 100$  Hz,  $E^* = 200$  MPa,  $l = 4 \cdot 10^{-2}$  m,  $h = 1.5 \cdot 10^{-2}$  m. The curve looks like

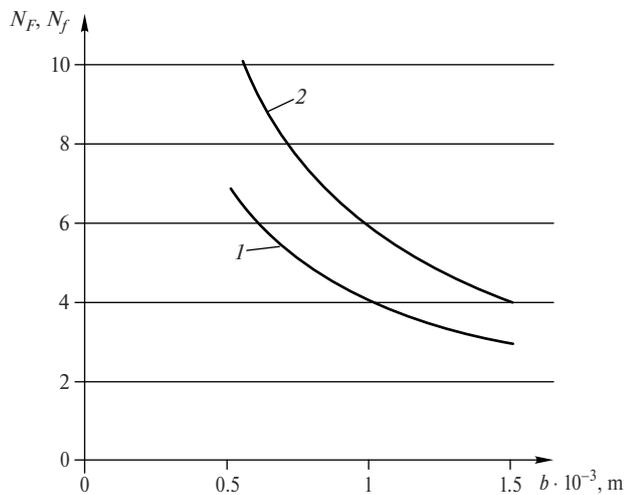


Fig. 2. Characteristics of the dependence of the number of plates in the packet of the connection node

$$N_f = 5.264 \frac{1}{b_1},$$

where the plate thickness indicator  $b_1 = [0.5; 1.0; 1.5]$ .

When plotting the graph, the number of plates in the package is also rounded to the nearest integer, and the calculated values are  $N_f = [4; 6; 11]$  accordingly.

The method for determining the number of plates in a package, taking into account the relative position of the graphs in Fig. 2, as well as expressions (8, 10), can be represented in the form of the following stages.

*Stage 1* – construction of the dependence  $N_f$ , which meets the criterion of rigidity, which ensures the operation of the vibration unit in the program mode.

*Stage 2* – construction of the  $N_F$  dependence, which determines the practical feasibility of the real creation of a connection unit that is dangerous due to the conditions of fatigue strength.

*Stage 3* – comparison and analysis of the ratio of the characteristics of the number of plates in the connection node for the adopted thickness ( $b$ ). In this case, inequality must be fulfilled

$$N_f(b) \geq N_F(b).$$

This point of the technique guarantees that the connection node will not be destroyed by fatigue during the operation of the vibration machine.

However, the final answer to the question of the number of plates in the connection node is provided only by the method of laboratory experiment. It is carried out for the specified ranges of change: the operating frequency of the vibrating machine; modulus of elasticity of the material and thickness of the plate; length and width of lintels; the magnitude of the forcing force in the working direction of the unit.

**Implementation example.** As an example of laboratory implementation, a two-coordinate vibration stand with excitation axes  $Oz$  (vertical) and  $Oy$  (horizontal) is considered, the connection nodes of which are made in the form of packages of flat elastic elements. The appearance of the horizontal coordinate package  $Oy$ , made of 10 plates, is shown in Fig. 3.

The following structural and mechanical parameters are adopted: plate material – PTK textolite with a thickness of  $b = (1 \cdot 10^{-3}; 1.5 \cdot 10^{-3})$  m, modulus of elasticity –  $E^* = 2 \cdot 10^8$  Pa; Jumpers have a length of  $l = 4 \cdot 10^{-2}$  m and width  $h = 1.5 \cdot 10^{-2}$  m.

Experimental studies on the stiffness characteristics of packages (shown in Fig. 4) each consisting of 10 plates were carried out.

In this case, the lower washer in the form of a disc is closed on a fixed table. The power load under the conditions of the experiment is applied to the upper corners in accordance with

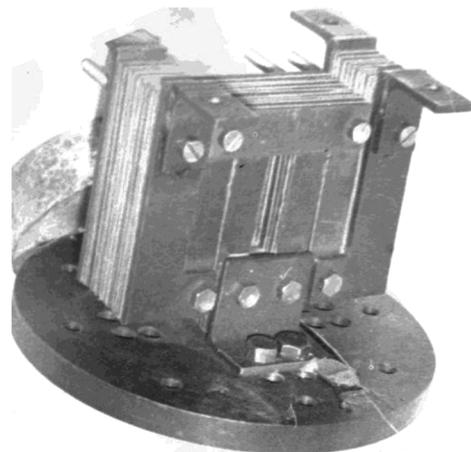


Fig. 3. Appearance of the flat element package

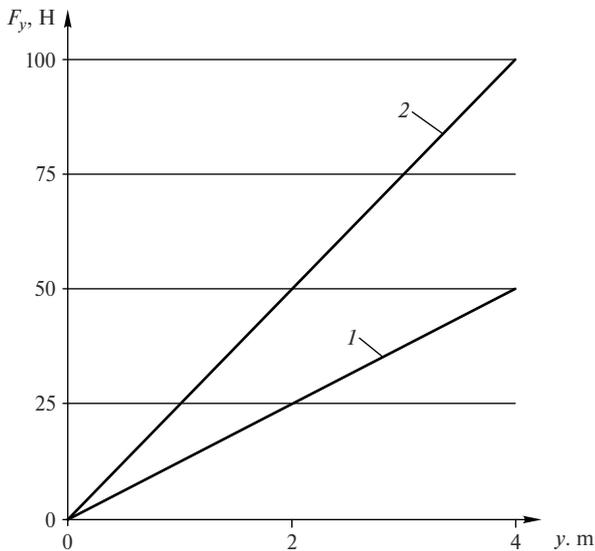


Fig. 4. Stiffness characteristics of flat bags elastic elements

the load on the graph of Fig. 2. Here, characteristic 1 corresponds to the thickness  $b = 1 \cdot 10^{-3}$  m, and the dependence of 2 thickness  $b = 1.5 \cdot 10^{-3}$  m.

Analysis of the behavior of the graphs shows their linearity, which excludes the appearance of super(sub) harmonics in the oscillations transmitted through the connection node. Also, the linearity of the transverse stiffness characteristics ensures the absence of mutual influences between the coupled coordinates in the mode of static and dynamic deformations of the plate elements of the package. As a result, the accuracy of multi-coordinate vibration reproduction on the stand platform increases, i.e. the reliability of mechanical tests of objects of spatial structure for strength and reliability.

**Conclusions.** The paper discusses the design features, mathematical model and calculation methods of elements of vibration machines for testing objects of spatial structure. Theoretical and experimental studies of vibration transmission units of installations for testing objects of spatial structure for multicomponent vibration made it possible to develop and propose a new type of packages of plate elastic elements. At the same time, the structurally elastic element is represented by a plate divided by straight internal slots.

It is shown that the external and internal bridges formed with the identity in width and length provide both the linearity of the transverse stiffness of the packages and the absence of interrelations between the conjugate coordinates in the static deformation mode. In contrast to elliptical, rod, electromagnetic, as well as lapped and lubricated joints, the joints considered in this work, made in the form of packages of elastic elements, are distinguished by: ease of manufacture; lack of additional energy to maintain the operability of the unit; low weight, and most importantly – the linearity of the stiffness characteristics in the coupled directions. The dependencies of the stiffness of the elastic element in the direction of transmitted vibration on the modulus of elasticity of the plate material, the area of an individual lintel, its length, width, and thickness have also been established. The limiting values of the forces of vibration exciters, which satisfy the conditions of stability of the plate package, are obtained.

A study on plate packet nodes for connecting vibrating exciter tables with a platform, which have specified values of longitudinal stiffness in the direction of transmitted vibration, as well as the resonance frequency of a package of plates, is presented. All this together makes it possible to increase the accuracy and reliability of mechanical tests of objects of spatial structure for multicomponent translational vibration. The practical significance of the above results lies in the creation of two, three-coordinate translational vibration stands for testing

for vibration resistance, vibration resistance, as well as vibration diagnostics of technical objects operated under the conditions of spatial vibration.

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## Пружні, частотні та характеристики стійкості пластинчастих вузлів з'єднання вібраційних машин

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**Мета.** Формалізувати конструкцію пружного елемента вузла з'єднання багатоосової вібраційної машини пластиною у вигляді внутрішніх вирізів і автономних перемичок. Отримати залежність жорсткості пластинчастого пружного елемента в напрямку вібрації, що передається, від модуля пружності матеріалу пластини, площі перерізу окремої перемички, її довжини, ширини й товщини. Визначити граничні значення змушуючих сил машини, що задовольняють умовам стійкості перемичок пакета пластин за Ейлером.

**Методика.** Під час створення конфігурації пружного елемента у вигляді поздовжніх перемичок парної кількості й моделювання характеристики жорсткості вузла з'єднання використані методи прикладної механіки й теорії опору матеріалів.

**Результати.** Базовими результатами роботи є математичне й фізичне моделювання характеристик пружності, стійкості та взаємовпливів у робочому та сполученому напрямках стосовно вузлів з'єднання багатокординатних вібраційних машин, що реалізовані через плоскі ба-

гатосекційні пластинчасті елементи. У сукупності це дозволяє створити енергоефективні вібраційні машини, підвищити точність програмного руху їх робочого органу, що у свою чергу збільшує, при видобутку корисних копалин – продуктивність гірничорудної техніки, а при механічних випробуваннях – їх достовірність.

**Наукова новизна.** Полягає в тому, що вперше отримана залежність жорсткості пластинчастого пружного елемента в напрямку вібрації, що передається, від модуля пружності матеріалу пластини, площі перерізу окремої перемички, її довжини, ширини й товщини. Показано, що жорсткість пластини у сполученому напрямку прямо пропорційна модулю пружності матеріалу, ширині перемички, її товщині в кубі та обернено пропорційна довжині перемички у третьому ступені. Встановлено, що для забезпечення умов стійкості перемичок пакета пластин стосовно граничних значень сил віброзбудників машини існує їх прямо пропорційна залежність від кількості пластин у пакеті, модуля пружності матеріалу, ширини перемички й товщини пластини у третьому ступені, і обернено пропорційна від довжини перемички у квадраті.

**Практична значимість.** Уперше запропонована конструктивна механічна модель пружного елемента, що представлений пластиною, поділеною внутрішніми прямими прорізами на ідентичні за шириною й довжиною зовнішні та внутрішні перемички. Створено реальний пластинчастий вузол з'єднання, наведені його експериментальні характеристики жорсткості в залежності від кількості пластин у пакеті.

**Ключові слова:** вібраційна машина, пластинчастий пружний елемент, жорсткість, стійкість, діапазон робочих частот

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