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DEFORMATION OF THE LONG CASING STRING ON CENTRALIZERS IN THE PROCESS OF ITS INSTALLATION IN A HORIZONTAL WELL

Purpose. Development of a method for calculating parameters of a stress-deformed state of compressed casing pushed into a horizontal well during its construction.

Methodology. The problem solution presupposes integrating the differential equation of the longitudinal bend of a long rod under its own weight. In the first approximation, reactions on the center-axis bearings are found disregarding the axial forces of friction. Our system of algebraic equations determines the deformation and force parameters at the sections between the supports based on the equations of the compatibility of the deformations and the equilibrium of the bending moments in the support sections.

Findings. A general solution of the basic differential equation of deformations of a horizontal pipe column is found taking into account friction and axial forces acting during its lowering. It forms the basis for calculating bucklings, rotation angles of rod intersections, its internal bending moments and transverse forces at the sections between the supports. It also makes allowance for additional moments of frictional forces acting on the centralizers. The solution of the problem for the reverse motion of the casing column is found.

Originality. The equation of connection between transverse and longitudinal forces in a long rod and reactions and forces of friction on supports is offered. The system of equations takes into account the equations of transverse forces, which allow determining the axial compressive forces simultaneously. The linearization method of the system of algebraic equations and its iterative solution with high accuracy is developed.

Practical value. The obtained results take into account the requirements of the construction technology of a horizontal well. Formulas for calculating the optimal distance between the centralizers are derived. The influence of deviations of the well direction from the horizontal on the change of the stress-deformed state of the casing which allows oilmen to increase its reliability and durability is considered.

Keywords: *horizontal well, casing string, flexible rod, longitudinal buckling, friction force, axial compression*

Introduction. Horizontal drilling is used to achieve productive subsurface layers in the construction of modern oil and gas wells [1, 2]. The column of steel casing pipes is lowered into the well from the mouth to the bottom and the annular space is cemented in accordance with the requirements of construction technology. This column rests on centralizers ensuring the coherence of the well pipes and walls, preventing their contact and forming a cement ring of the same thickness and strength [3]. The centralizers are made of curved steel strips shaped as a rotational ellipsoid and fixed regularly on pipe columns [4].

The weight of boring casing facilitates their running into the vertical, inclined and curved wells. However, to push the column into the canopy and horizontal areas, additional effort is needed and energy is needed to overcome frictional forces.

Literature review. The literature concerned with mechanical deformations of long casing strings in horizontal wells is quite diverse. Many researchers have dealt with different aspects of pipe buckling in horizontal wellbores.

Rachkevich R. [1] explored drill pipe columns compressed in a horizontal well under their own weight. The stress-strain state of a drill string during its compression in a horizontal borehole is considered. The function of the elastic axis of the drill string as a wave curve is obtained, which enables one to improve the analytical models: evaluation of decrease in axial force depending on the drill string elastic axis shape, determination of the force of pressing of the string to borehole walls, and calculation of normal stresses in the cross section of drill pipes.

However, he examined only the section of the pipes with a significant axial force, a sinusoidally buckled column, and its ability to lean against the opposite walls every semiperiod. The problem solution is constrained by the flat bend shape and disregards friction and substantial torque required for drilling.

Gulyayev V. and Glazunov S. [2] analyzed the behavior of a real drill column rotating in the channel of a horizontal well. They formulate a problem of the bifurcation buckling and small bending vibrations of a rotating drill string lying in a horizontal wellbore. With allowance for friction forces and additional constraints, differential equations have been derived, for which solutions of eigenvalue problems of free vibrations of drill strings of finite and unlimited length have been constructed.

These papers [2] took into account the equation of weight force equilibrium, wall behavior, friction, compression forces and their torques and obtained a solvable homogeneous differential equation of the fourth order for bending deformations of a compressed rod, which rests on the wall of a cylindrical channel and rotates. Their research solved the problems dealing with axial force eigenvalues critical for the rod stability loss, as well as the rotational frequencies critical for its bending oscillations. In this case, deformation and power parameters of the stress-strain state of the horizontal drill column were not determined.

Vytvytskyi I., et al. [3] formulated the problem of rational centering of the casing under the complex configuration of well axis using elastic-rigid centralizers. The model of the “casing-well” system is under consideration. In the simulation system, the column in the hole is loaded by two sets of forces: the forces of gravity, distributed along the axis of the pipe and pressure forces caused by the complex configuration of the well axis and distributed between the centering devices. The influence of structural peculiarities of the cyclically symmetric “bow” type centralizers on their rigidity and toughness characteristics has been taken into account.

According to the authors [3], the obtained results provide for optimizing the exact number and intervals of equipping the casing with centering devices and, thereby, avoiding the formation of dead zones in the annulus. This will give the opportunity to achieve high-quality cement job and make casing of the well of any configuration reliable and time-proof.

Shatskyi I., et al. considered the problem of the interaction of elastic rod casing centralizers with the wellbore wall [4]. The paper aims at studying the influence of the axial mobility of the centralizer's ends on the parameters of its rigidity and strength. These characteristics are necessary to assess the possibility of the casing string and quality of well completions.

Gao G. and Miska S. [5] studied the sinusoidal postbuckling configuration of a long pipe constrained in a horizontal well. In this paper, the buckling equation and natural boundary conditions are derived with the aid of calculus of variations. The natural and geometric boundary conditions are used to determine the proper solution that represents the postbuckling configuration. Effects of friction and boundary conditions on the critical load of helical buckling are investigated. Friction can increase the critical load of helical buckling significantly. Analytical solutions for both sinusoidal and helical post-buckling configurations are derived, and a practical procedure for modeling of axial load transfer is proposed.

Mitchell R. F. described analytical solutions for pipe buckling in vertical and horizontal wells [6] and lateral buckling of pipe with connectors in horizontal wells [7].

Researchpaper [6] presents two new analytic solutions for the horizontal well problem. These analytic solutions are valuable both for predicting previously unanticipated buckling behavior and for providing guidance in further numerical evaluations of this problem. Buckling length change calculations are determined analytically, and pipe curvature, bending moment, and bending stresses are evaluated. The contact loads between tubing and wellbore are determined, then used to limit the range of validity of the solutions. The critical force for helical buckling is determined for horizontal wells.

Article [7] indicate that bending stresses are greater because of connector standoff. Laterally buckled pipe with connectors is analyzed for the first time here. It presents an analytic solution of the beam-column equations in a horizontal wellbore with pipe weight. Pipe deflections, contact loads, and bending stresses are determined with explicit formulas. Sag between connectors is calculated so that pipe body contact with the wellbore between connectors can be determined.

Ren Fushen, et al. [8] focused on differential equation of the dynamic buckling and equations of the critical load of the sinusoidal buckling and the spiral buckling. This paper focuses on the rotational drill string in horizontal well. The differential equation of the dynamic buckling is established considering some various factors, including friction, the drill string-borehole interaction, and rotation, and the equations of the critical load of the sinusoidal buckling and the spiral buckling are derived.

In 1996, J. H. Sampaio presented an innovative mathematical model for mechanical buckling of drillstrings within curved bore-holes in his PhD dissertation. Unlike the drill pipe, the casing is run down without being rotated. When centralizers are not installed, frictional forces distributed along its entire length of the column increase quite heavily affecting it when moving further away from the free end. In this case, there is a complex longitudinal bend of the rod in the space of the horizontal cylindrical canal.

Miller J., et al. [9] thoroughly investigated this phenomenon using Kirchhoff's model for elastic rods, supplemented by its frictional interaction with channel walls, its computer realization and field simulation.

The researchers proved that the rod depends on the axial force increase and the distance from the free end: moving farther away, the rod is sinusoidally buckled with the period decreasing and amplitude increasing. As the axial force of the corresponding supercritical value reaches the long rod, it rotates into the spiral line touching the cylindrical surface. When the rotation number exceeds a certain value, the rod blocks the channel and stops its further movement.

To prevent this self-locking in sloping and horizontal parts of the well, casing is installed on centralizers serving as supports, and when moving, they act like stripes to reduce friction with the well walls.

Our previous research [10] resulted in the differential equation of deformations of the casing column as an elastic rod under its own weight action in the section between two adjacent centralizers arbitrarily oriented regarding the vertical

$$\theta'' - t_0(\theta - \theta_0) + u_0 + j(s - s_0)(\sin\vartheta_n + \theta\cos\vartheta_n) = 0, \quad (1)$$

where θ is the angle between the tangent to the rod's curved axis in the intersection with the coordinate s and the local axis passing through the two supports; ϑ_n is the anti-zenith angle to the vertical of this local axis of the n^{th} section inclination; j is the rod's specific gravity; s_0 is the coordinate of the rod's intersection in which the following initial parameters are given: θ_0 – the inclination angle of the tangent to the local axis; t_0 – axial force; u_0 – transverse force.

In equation (1), all force factors (weight j, forces t and u, moments q) are divided by stiffness EJ (E – material elasticity, J – the transverse inertia moment). This is based on the fact that bending deformations of the casing column should not exceed elasticity limit and be proportional to bending moments with the coefficient EJ [2]. Thus, preserving the solution universality, one can investigate elastic deformations of a long rod with a single stiffness on its bend. Besides, the bending moment equals the rod curvature.

Unsolved aspects of the problem. It is necessary to determine geometrical and power parameters of casing deformations when the column slowly advances into a horizontal well. The column tube is a long, indistinguishable rod supported by centralizers (equally spaced hinges) to keep casing centered in a wellbore. Weight distribution along the pipe length makes its sections between the supports bend. Friction forces arise on centralizers as the column advances and causes axial compression forces and longitudinal buckling in the pipe body, thus increasing their deformations. The force magnitude varies at different segments and increases with distance from the free end.

Fig. 1 visualizes forces affecting casing design on centralizers. The notation expresses the following: $n - 1$, n, $n + 1$ – numbers of segments and supports; l_n, l_{n+1} – sections lengths between the supports; j – weight distribution on casing pipes; R_{n-1}, R_n, R_{n+1} – reactions on supports; F_{n-1}, F_n, F_{n+1} – frictional forces on supports; t_{n-1}, t_{n+2} – axial compression forces.

The section numbering of the horizontal pipe starts from its free end: the last segment has number *1*, the second last one has number *2* and so on. The support in the initial intersection of each section (where $s_0 = 0$) has a segment number. The coordinate s is the initial intersection that sets the direction of the column's free end, with the support number *0*.

Purpose. The objective of this article is to develop a method for calculating parameters of an intense-deformed state of a long horizontal indistinguishable rod in the sections between the supports under the simultaneous action of the longitudinal force and uniformly distributed transverse load. To do this, we need to solve the differential equation of longitudinal bend (1) taking into account the rod's axial compression force.

For all sections of the horizontal well, $\vartheta_n = 90°$; then the basic equation (1) will have the following form

$$\theta'' - t_n(\theta - \theta_n) + u_n + js = 0, \quad (2)$$

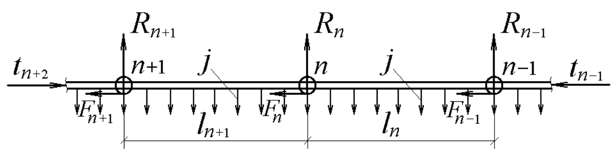

Fig. 1. Scheme of forces acting on the casing, supported on the centralizer

where θ_n is the inclination angle of the tangent to the horizontal; t_n is axial force; u_n is transverse force; q_n is bending moment.

Fig. 2 shows all these variables in the calculation diagram of the equilibrium of forces and moments in the casing section. Here, θ, t, u, q are, respectively, the same parameters at a distance s from the initial intersection along the rod's curved axis; z_n is the local axis passing through the two supports of the n^{th} section.

Equation (2) is the integral of the differential equation of the longitudinal bend of the rod with account of its own weight [1]. Its solution specifies the functions of the core buckling $x(s)=\int\theta ds$; angles of crossings $\theta = \theta(s)$; internal bending moments $q(s) = d\theta/ds = \theta'$ (this is also the rod curvature) and internal transverse forces $u(s) = -dq/ds = -q'$ [10].

Solving the problem: the first approximation. First, we believe that friction might be disregarded. Consequently, the rod's axial compression forces are not available. The basic equation (2) is simplified

$$\theta'' = -js - u_n.$$

Its integral is

$$x=\int\theta ds=-j\frac{s^4}{24}-u_n\frac{s^3}{6}+\alpha\frac{s^2}{2}+\beta s+\gamma,$$

where α, β, γ are integration constants.

No buckling on the supports provides the following boundary conditions: $x(0) = x(l) = 0$ (where l is segment length), which helps to find u_n and γ. Hence, we obtain the functions of rod buckling x, rotation angles θ, bending moments q, and transverse forces u.

$$x(s)=-\frac{j}{24}\left(s^4-ls^3\right)-\frac{\alpha}{2l}\left(s^3-ls^2\right)-\frac{\beta}{l^2}\left(s^3-l^2s\right); \quad (3)$$

$$\theta(s)=-\frac{j}{24}\left(4s^3-3ls^2\right)-\frac{\alpha}{2l}\left(3s^2-2ls\right)-\frac{\beta}{l^2}\left(3s^2-l^2\right); \quad (4)$$

$$q(s)=-\frac{j}{4}\left(2s^2-ls\right)-\frac{\alpha}{l}\left(3s-l\right)-6\frac{\beta}{l^2}s; \quad (5)$$

$$u(s)=-j\left(s-\frac{l}{4}\right)+3\frac{\alpha}{l}+6\frac{\beta}{l^2}. \quad (6)$$

We might derive unknown coefficients α and β for each segment from the system of linear equations for the boundary conditions at segment edges (on supports). For the indistinguishable rod, such conditions include deformation compatibility and bending moments equilibrium. Hence, considering rotation angles equality (4) and internal moments (5) in the support intersections of adjacent sections, we have

$$\theta_{n+1}(l_{n+1}) - \theta_n(0) = 0; \quad (7)$$

$$q_{n+1}(l_{n+1}) - q_n(0) = 0. \quad (8)$$

Expression (5) generates the first equation of the system provided there is no bending moment at the pipe's free end: $q_1(l_1) = 0$. Consequently, bucklings, bending angles and bending moments in the first section will be larger than those in farther sections. When removed from the free end, its effect extinguishes, the segments are balanced and interact in the connected intersections under similar conditions. Due to the same length, their stress-strain is symmetrical regarding the mid-point of the run between the supports. Therefore, there are no rotations in the support sections. The last equation closing the linear system is obtained from expression (4) for the selected most-distant segment $(n + 1)$ if $q_{n+1}(0) = 0$.

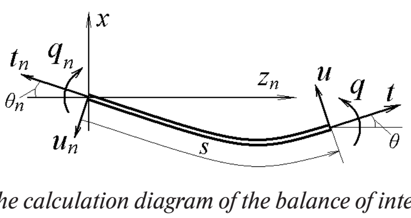

Fig. 2. The calculation diagram of the balance of internal forces and moments on the pipe section (the distributed weight j is not shown)

For the numerical evaluation of the desired parameters of the problem, a system of 10 linear equations for five sections was compiled and solved by programming in Mathcad. For this purpose, expressions (4, 5) were used under conditions (7, 8) for supports 1–4 and the first and last equations. The results showed that the moment on support 4 differs from that on support 5 only by 0.56 %. Consequently, with this error, we can assume that already the fifth section has symmetric deflections.

With a symmetric buckling pattern, the support moment is

$$q_n(0)=q_n(l)=-\frac{jl^2}{12}, \quad (9)$$

while the largest curvature radius of the rod's buckling is

$$x_{\max}=x(l/2)=-\frac{jl^4}{384}. \quad (10)$$

According to construction technology requirements, the pipe's maximum deflection should be limited to avoid its friction on well walls and guarantee a clearance for high-quality cementing. Then, restrictions, for example $x_{\max} \le l/1000$, might determine permissible distance between supports: $l_{\max}\le\sqrt[3]{0.384/j}$.

Our calculations prove that with the same distance between the supports in the final section buckling is 2.5 times larger than the allowance as there is no bending moment at the free end. Nonetheless, this buckling immediately decreases by 8 %, if the final section length is reduced to

$$l_1=l\sqrt{2/3}=0.8165l. \quad (11)$$

In this case, the bending moment on support 1 equals moment (9)

$$q_1(0)=-jl_1^2/8=q_2(l)=-jl^2/12,$$

and all the sections, starting with number two, have a symmetric buckling pattern regardless of the number involved in the system of equations.

At the free end, the support behavior coincides with the transverse force

$$R_0 = u_1(l_1) = 3jl_1/8. \quad (12)$$

The reaction of any intermediate support causes difference (increase) of internal transverse forces in the joint cross section of two adjacent sections

$$R_n = u_{n+1}(l_{n+1}) - u_n(0). \quad (13)$$

We established that the final section length reduction (11) when the symmetric buckling pattern is established in all segments ensures that the responses on all the supports equal the section weight: $R_n = jl$ (except for support 1, where it is more than 1 %).

The same equation system is solved for reduced length $l_1 = 0.8l$. Besides, it was found that on the final support the reaction is less than the calculated one (12) only by 0.72 %; on support 2, the reaction differs from the weight of the section by 0.26 %; on subsequent supports reactions equal the sections weight (the difference is less than 0.07 % and disappears after the fifth one).

Thus, the solution to the problem of deforming a long horizontal rod on supports without friction and axial forces can be found in closed form (3–6). Due to this, reactions on supports become known in the first approximation.

Axial forces mechanism. As the pipe string advances in a horizontal well, axial forces arise only due to friction forces on the supports, and in the area between the supports the axial force in the pipe body remains constant.

Let us establish the connection between the support reaction, transverse force and axial force in a long rod. For this, we apply a linear model of the friction force F, which is proportional to the support reaction R_n of the n^{th} section: $F_n = k_t R_n$, where k_t is friction coefficient.

At the free end, the reaction of support (12) exerts friction force and axial compression force t_1 in the 1st section

$$t_1 = k_t R_0 = k_t \cdot u_1(l_1). \quad (14)$$

The reaction on support 1 according to balance (13) will be

$$R_1 = u_2(l_2) - u_1(0).$$

The difference in internal transverse forces on the opposite edges of any pipe part is equal to its weight, regardless of the stress-strain state

$$u_n(l_n) - u_n(0) = jl_n. \quad (15)$$

Then axial force in segment 2, exerted by frictional force on support 1 and axial force from the edge of segment 1, is

$$t_2 = k_t R_1 = k_t(u_2(l_2) + jl_1).$$

The reaction of support 2, the weight of section 2 and axial force in section 3 are, respectively,

$$R_2 = u_3(l_3) - u_2(0);$$

$$u_2(l_2) - u_2(0) = jl_2;$$

$$t_3 = k_t R_2 + t_2 = k_t(u_3(l_3) + j(l_2 + l_1)).$$

In general, in the n^{th} section, the axial force t_n is caused by friction force of on support $(n - 1)$ and axial force t_{n-1} acting on the adjacent segment $(n - 1)$ (closer to the free end). Therefore, the response to support $(n - 1)$, weight of the segment $(n - 1)$ and the axial force in the n^{th} section are respectively

$$R_{n-1} = u_n(l_n) - u_{n-1}(0);$$

$$u_{n-1}(l_{n-1}) - u_{n-1}(0) = jl_{n-1};$$

$$t_n = k_t R_{n-1} + t_{n-1} = k_t(u_n(l_n) + jL_{n-1}), \quad (16)$$

where $L_{n-1} = \sum_{i=1}^{n-1} l_i$ is the length of the pipe column from support $(n - 1)$ to the free end.

These equations express the relation (equilibrium) between external and internal forces in an arbitrary n^{th} region and on the support $(n - 1)$.

Consequently, when casing is lowered into a horizontal well, the axial force in the pipe body between the supports determines the column section weight at its free end and the value of lateral force at the section end. For a frictionless case, this force is calculated using (6). When frictional forces are obvious, the rod experiences a longitudinal – beside transverse – bend. Therefore, we observe a different value of the transverse force. The key to its solution is basic equation (2).

The solution of the basic equation of deformation. When the column moves, its pipes are compressed, and the axial force is negative $t < 0$. If $|t| = \tau^2$, then we will take into consideration its sign in the equation of deformation (2) and get

$$\theta'' + \tau^2\theta = -js + \tau^2\theta_n - u_n. \quad (17)$$

The general solution of this linear nonhomogeneous differential equation of the 2^{nd} order is quite understandable

$$\theta = \alpha\cos\tau s + \beta\sin\tau s - \frac{js}{\tau^2} + \theta_n - \frac{u_n}{\tau^2}.$$

Hence, the rod buckling in the section between the supports is

$$x = \int\theta ds = \alpha\frac{\sin\tau s}{\tau} - \beta\frac{\cos\tau s}{\tau} - \frac{js^2}{2\tau^2} + \left(\theta_n - \frac{u_n}{\tau^2}\right)s + \gamma.$$

Boundary conditions on supports $x(0) = x(l) = 0$ make it possible to determine integration constants $(\theta_n - u_n/\tau^2)$ and γ, as well as the rod buckling x, rotation angles θ, bending moments q, and transverse forces u, respectively

$$x(s) = \alpha s\left(\frac{\sin\tau s}{\tau s} - \frac{\sin l\tau}{l\tau}\right) + \beta s\left(\frac{1-\cos\tau s}{\tau s} - \frac{1-\cos l\tau}{l\tau}\right) + \frac{j}{\tau^2}\cdot\frac{s(l-s)}{2}; \quad (18)$$

$$\theta(s) = x' = \alpha\left(\cos\tau s - \frac{\sin l\tau}{l\tau}\right) + \beta\left(\sin\tau s - \frac{1-\cos l\tau}{l\tau}\right) + \frac{j}{\tau^2}\left(\frac{l}{2} - s\right); \quad (19)$$

$$q(s) = \theta' = -\alpha\tau\sin\tau s + \beta\tau\cos\tau s - \frac{j}{\tau^2}; \quad (20)$$

$$u(s) = -q' = \alpha\tau^2\cos\tau s + \beta\tau^2\sin\tau s. \quad (21)$$

Expansion of the system of equilibrium equations to account for axial forces. According to the boundary conditions at the edges of the segments (on the supports), it is necessary to make a system of equilibrium equations which will generate unknown integration constants.

According to (20), for the final support, on which the free end of section 1 was backed, the following equation can be made

$$q_1(l_1) = -\alpha_1\frac{(l\tau)_1\sin(l\tau)_1}{l_1} + \beta_1\frac{(l\tau)_1\cos(l\tau)_1}{l_1} - \frac{j}{t_1} = 0. \quad (22)$$

Here and in the following equations we use intentionally formed parameters $(l\tau)_n$, which also occur as arguments of trigonometric functions.

According to (19), we offer one equation of deformation compatibility (7) for each n^{th} support, where

$$\theta_{n+1}(l_{n+1}) = \alpha_{n+1}\left(\cos(l\tau)_{n+1} - \frac{\sin(l\tau)_{n+1}}{(l\tau)_{n+1}}\right) + \beta_{n+1}\left(\sin(l\tau)_{n+1} - \frac{1-\cos(l\tau)_{n+1}}{(l\tau)_{n+1}}\right) - \frac{jl_{n+1}}{2t_{n+1}};$$

$$\theta_n(0) = \alpha_n\left(1 - \frac{\sin(l\tau)_n}{(l\tau)_n}\right) - \beta_n\frac{1-\cos(l\tau)_n}{(l\tau)_n} + \frac{jl_n}{2t_n}.$$

According to (20), we can make another equation – support point equilibrium (8), in which

$$q_{n+1}(l_{n+1}) = -\alpha_{n+1}\frac{(l\tau)_{n+1}\sin(l\tau)_{n+1}}{l_{n+1}} + \beta_{n+1}\frac{(l\tau)_{n+1}\cos(l\tau)_{n+1}}{l_{n+1}} - \frac{j}{t_{n+1}};$$

$$q_n(0) = \beta_n\frac{(l\tau)_n}{l_n} - \frac{j}{t_n}.$$

All these equations contain the parameter $1/t_n = 1/\tau_n^2$ which is inverse to axial force magnitude that must be determined from the system of equations. Thus, for each rod section, three unknowns must be identified: α_n, β_n, $1/t_n$. To calculate them, we use the third equation obtained due to the relation established between its friction force, reaction on the support, transverse force and axial force in the rod.

To support the free end in accordance with (14)

$$u_1(l_1)=\frac{\tau_1^2}{k_t},$$

therefore, an additional equation can be offered for segment *1* in accordance with (21)

$$\frac{u_1(l_1)}{\tau_1^2}=\alpha_1\cos(l\tau)_1+\beta_1\sin(l\tau)_1=\frac{1}{k_t}.$$

For any n^{th} section in accordance with (16)

$$u_n(l_n)=\frac{t_n}{k_t}-jL_{n-1},$$

therefore, according to (21), we can make the third equation

$$\frac{u_n(l_n)}{\tau_n^2}=\alpha_n\cos(l\tau)_n+\beta_n\sin(l\tau)_n+\frac{jL_{n-1}}{t_n}=\frac{1}{k_t}.$$

Obviously enough, the previous two equations are not sufficient for section 1; therefore, our next equality will close the system. For example, for the beginning of section ($n+1$), it will account for fixed end $\theta_{n+1}(0)=0$ or free restraint $q_{n+1}(0)=0$. Moreover, the nonlinearity of solution (21) makes it applicable to the weight of segment $(n+1)^{th}$ under condition (15). Hence, we form the last equation of the system

$$-\alpha_{n+1}\left(1-\cos(l\tau)_{n+1}\right)+\beta_{n+1}\sin(l\tau)_{n+1}-\frac{j\,l_{n+1}}{t_{n+1}}=0.$$

General methodology of solving the problem. The resulting system of algebraic equations is nonlinear. It contains the function of an unknown parameter $l\tau=l\sqrt{t}$ depending on the axial force t in each segment which in turn is to be determined from the system as $1/t$.

The proposed model for the axial force formation in a rod makes the problem solution quite possible. Substituting axial force (16) in basic equation (17), we have

$$\theta''+k_t(u_n(l_n)+jL_{n-1})\theta=-js+t_n\theta_n-u_n. \qquad (23)$$

As the length l_n and distance L_n from the segments to their free ends are known, after the rod deformation the boundary parameters $u_n(l_n)$, u_n, θ_n, t_n get their specific constant values.

Parameter k_t in equation (23) projects its solution as it is the coefficient of friction and can take values within $0\le k_t\le k_{\max}<1$ and results in a function describing the rod's stress-strain state in the area between the supports. The solution of the basic equation with $k_t=0$ (in non-friction cases) takes the closed form (3–6) and serves as the first approximation.

We apply Poincare's theorem on the continuous dependence of a differential equation solution on a parameter. With a gradual minor increase in the parameter, equation (23) with a nonzero k_t will slightly deviate from the found solution of the first approximation. Thus, when axial forces ($0<k_t<1$) are insignificant, the solutions of the basic equation (namely, the function of the buckling distributions, bending angles, bending moments and transverse forces on the rod sections) will not change dramatically (will not jerk), as its deviation is minimal from the solution (3–6).

Proceeding from this, the following methodology for problem solving was developed.

Initially, we find the transverse forces $u_n(l_n)$ at the end of each section in the first approximation. With regular placement of supports – this is $u_n(l)=jl/2$, and for the free end – (12). Where the lengths l_n of each segment are different, it is necessary to compile and solve the corresponding equations system of deformation compatibility and support points equilibrium (7, 8) for the same number and length of the sections, which provides finding of all values of $u_n(l_n)$.

Then, applying formula (16) to each section, we calculate axial forces t_n and determine values of $(l\tau)_n=l_n\sqrt{t_n}$ as known. Their substitution into the system of algebraic equations transforms the functions of parameters $(l\tau)_n$ into known coefficients, and the system itself becomes linear regarding unknown α_n, β_n, $1/t_n$.

The solution of this system gives more accurate values of t_n and parameters $(l\tau)_n$ in the next approximation. The substitution of the latter into the system of equations and its solution should be repeated until the desired value accuracy of axial forces t_n is achieved.

Calculation analysis. Our algorithm is implemented in MatLab software environment. Determination of all parameters of the stress-strain state in 100 segments of a rod (a system of 300 equations), primarily axial forces with an accuracy of 14 digits, takes 386 iterations and 10 secs on a single core microprocessor Intel Pentium 4 with a working frequency of 2.6 GHz.

Fig. 3 presents calculation results of parameters of a stress-deformed state of a horizontal casing column with a diameter of 140 mm and a wall thickness of 10.5 mm ($l=12$; $k_t=0.5$; $j=1.85\cdot10^{-4}$ m^{-3}; $EJ=1.7\cdot10^6$ Nm^2).

They show that from the 2nd to the 100th run deformation and power parameters increase: maximum curvature and turns – by 66 %, support points – by 44 %, moments in the middle of the segment – by 78 %, axial forces – by 75 times. It should be noted that distribution of transverse forces varies from the linear pattern caused by a uniformly distributed transverse load to a sinusoidal configuration resulting from a significant static force and a longitudinal buckling of the run.

Accounting for technological conditions of a well construction. ***Constraints on maximum bucklings***. When the distance from the free end of the pipe increases, significant growth of its bucklings is unacceptable. However, axial force increase in each subsequent section is restricted and not larger than k_tR_n. With the regular placement of supports, transverse force is $u(l)=0.5jl$. Then according to (16), axial force will be

$$t_n=\tau^2=k_t(u(l)+jL_{n-1})=k_tjl(n-0.7), \qquad (24)$$

for the given length $l_1=0.8l$.

As it follows from (24), axial forces in adjacent areas differ by less than 5 % after 20 runs, and by less than 1 % – after 100 runs, respectively. With such an error, we can assume that the forces, as well as other parameters of the stress-strain state, are the same.

Substituting expressions of rotating angles (19) in $\theta_n(0)=\theta_n(l)$, we obtain

$$-\alpha\left(1-\cos l\tau\right)+\beta\sin l\tau=\frac{jl}{t}.$$

The same equation can be obtained using (15) expressing the weight of an arbitrary section.

The expression of transverse force (21) at the end of the segment gives the following equation

$$\frac{u(l)}{\tau^2}=\alpha\cos l\tau+\beta\sin l\tau=\frac{jl}{2t}.$$

The solution of the system of these two equations with respect to α and β and the expressions (18–21) shows deflections of the rod x, rotation angles θ, bending moments q, transverse forces u, respectively

$$x(s)=-\frac{jl^2}{2t}\cdot\frac{\sin l\tau}{l\tau}\left(\frac{\sin\tau s}{\sin l\tau}-\frac{1-\cos\tau s}{1-\cos l\tau}\right)+\frac{js(l-s)}{2t};$$

$$\theta(s)=-\frac{jl\cdot\sin l\tau}{2t}\left(\frac{\cos\tau s}{\sin l\tau}-\frac{\sin\tau s}{1-\cos l\tau}\right)+\frac{j(l-2s)}{2t};$$

$$q(s)=\frac{jl^2}{2}\cdot\frac{\sin l\tau}{l\tau}\left(\frac{\sin\tau s}{\sin l\tau}+\frac{\cos\tau s}{1-\cos l\tau}\right)+\frac{j}{t};$$

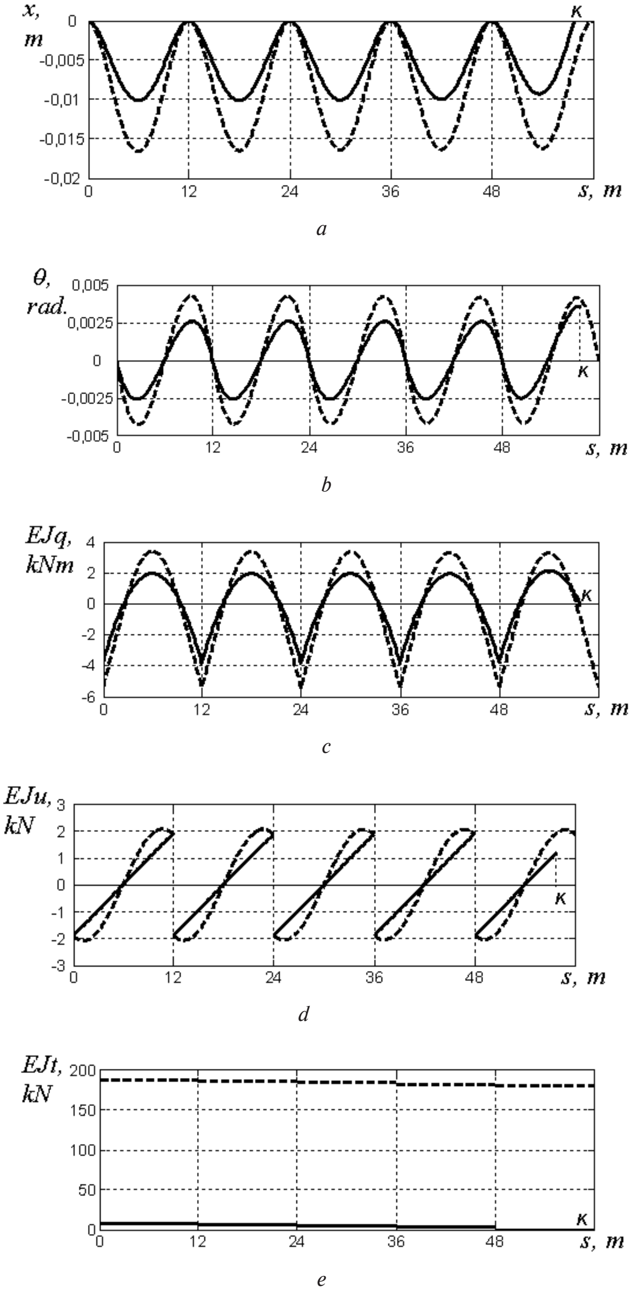

Fig. 3. Diagrams of the deformation and force parameters of the horizontal casing, combined for the first and last sections:

a – the buckling of the sections; b – the intersection angles; c – bending moments; d – transverse forces; e – axial compression forces; solid lines – segments from 5 to 1; dashed lines – segments from 100 to 96; K – free end section (support 0)

$$u(s) = -\frac{jl\cdot\sin l\tau}{2}\left(\frac{\cos\tau s}{\sin l\tau} - \frac{\sin\tau s}{1-\cos l\tau}\right).$$

As $\theta(l/2) = x'(l/2) = 0$, then the largest buckling in the middle of the segment is

$$x_{\max} = x(l/2) = -\frac{jl^4}{384}\left(\frac{3\operatorname{tg}(l\tau/4)}{(l\tau/4)^3} - \frac{3}{(l\tau/4)^2}\right). \quad (25)$$

Decomposition of the expression containing $\xi = l\tau/4$ in parentheses in a series gives

$$\frac{3}{\xi^2}\left(\frac{\operatorname{tg}\xi}{\xi} - 1\right) = 1 + \frac{2}{5}\xi^2 + \frac{17}{105}\xi^4 + \frac{62}{945}\xi^6 + \ldots.$$

With $t = 0$, this row equals 1, and the buckling fully coincides with (10). When $l\tau = 2\pi$, then the series and buckling grow indefinitely. Therefore, it is necessary to establish restrictions on the curvature growth of pipe bucklings; for example, their increase may not exceed 50 % (it may not be more than 1.5 times of its original value). To do this, we need to solve the transcendental equation derived from (25)

$$3\operatorname{tg}(l\tau/4) - 3(l\tau/4) = 1.5(l\tau/4)^3.$$

Its solution is $l\tau/4 = 0.911$. Expression (24) helps to find the number of runs with permissible length $n = (l\tau)^2/(k_t jl^3) + 0.7$. To restore the permissible buckling in the subsequent areas, the distance between supports should be reduced by 10 %, because $\sqrt[4]{1/1.5} = 0.9$.

Friction moments on centralizers. Friction force applied to the centralizer's outer surface with a diameter d exerts friction moment $m_n = k_t R_n d/2$ in the support cross section. Then, equation (22) for the ultimate support will have the following form

$$-\alpha_1(l\tau)_1\sin(l\tau)_1 + \beta_1(l\tau)_1\cos(l\tau)_1 - \frac{jl_1}{t_1} = -\frac{(l\tau)_1^2 d}{2l_1}.$$

The equation of the moment equilibrium (8) should be written as $q_{n+1}(l_{n+1}) - q_n(0) = -m_n$, where the moment friction is $m_n = k_t(u_{n+1}(l_{n+1}) - u_n(0))d/2$ (13).

Using expressions of transverse forces (21), we obtain

$$m_n = \frac{k_t d}{2}\left(-\alpha_n\frac{(l\tau)_n^2}{l_n^2} + \alpha_{n+1}\frac{(l\tau)_{n+1}^2\cos(l\tau)_{n+1}}{l_{n+1}^2} + \right.$$
$$\left. + \beta_{n+1}\frac{(l\tau)_{n+1}^2\sin(l\tau)_{n+1}}{l_{n+1}^2}\right).$$

Substituting this friction moment together with expressions (20) into the equation of moment equilibrium, it is necessary to group addends with the same coefficients α and β.

Deviation of the real well from the horizontal. Due to geological, technical and technological deviations during well drilling, individual sections may not coincide with the horizontal. As a result, casing supports have slight displacements vertically up or down, and the local axes of the sections between the supports are actually reciprocal turns, causing the basic bending moments to change. Therefore, at cross sections of such a rod, the equation of deformation compatibility should be the equality of anti-zenith angles at the end of the next (counting from the free end) and at the beginning of the previous sections $\vartheta_{n+1}(l_{n+1}) - \vartheta_n(0) = 0$.

To determine the actual profile of the well before installing the casing means to conduct its deep inclinometry – to measure the actual anti-zenith inclination angles of the section axes to the vertical (with a step of 10 m). Knowing these angles, we can determine the zenith angle of any section on the segment of the rod [10]: $\vartheta(s) = \vartheta_n + \theta(s)$, $\vartheta_{n+1}(l_{n+1}) = \vartheta_{n+1} + \theta_{n+1}(l_{n+1})$ and $\vartheta_n(0) = \vartheta_n + \theta_n(0)$. Consequently, instead of equation (7), the system needs to apply the equation of deformation compatibility

$$\theta_{n+1}(l_{n+1}) - \theta_n(0) = \vartheta_n - \vartheta_{n+1}.$$

Pipe column pulling case. Well construction might face two cases (scheduled and emergency) during which the pipe column moves in the reverse direction from the free end. In emergency cases, it is necessary to pull the column out of the well to have it repaired. The planned case is presupposed by the technology of pulling tension on the column string after its installation in a horizontal well to reduce pipe buckling and to form large gaps to be later cemented.

When pulling a tube column from the well, friction forces on the supports stretch the pipe; therefore, the axial force is positive. By denoting $t_n = \tau^2$ and using (2), we obtain the following differential equation

$$\theta'' - \tau^2\theta = -js - (u_n + \tau^2\theta_n).$$

Both the general solution of this equation and the integral contain hyperbolic functions

$$\theta = \alpha \operatorname{ch} \tau s + \beta \operatorname{sh} \tau s + \frac{j}{\tau^2} s + \frac{u_n}{\tau^2} + \theta_n;$$

$$x = \int \theta ds = \alpha \frac{\operatorname{sh} \tau s}{\tau} + \beta \frac{\operatorname{ch} \tau s}{\tau} + \frac{j}{\tau^2}\frac{s^2}{2} + \left(\frac{u_n}{\tau^2} + \theta_n\right) s + \gamma.$$

Under the boundary conditions on the supports $x(0) = x(l) = 0$, two coefficients can be obtained: $(\theta_n + u_n/\tau^2)$ and γ. Then we determine bucklings of the rod x, angles of rotation θ, bending moments q, and transverse forces u, respectively

$$x(s) = \alpha s\left(\frac{\operatorname{sh} \tau s}{\tau s} - \frac{\operatorname{sh} l\tau}{l\tau}\right) + \beta s\left(\frac{\operatorname{ch} \tau s - 1}{\tau s} - \frac{\operatorname{ch} l\tau - 1}{l\tau}\right) - \frac{j}{\tau^2}\cdot\frac{s(l-s)}{2};$$

$$\theta(s) = \alpha\left(\operatorname{ch} \tau s - \frac{\operatorname{sh} l\tau}{l\tau}\right) + \beta\left(\operatorname{sh} \tau s - \frac{\operatorname{ch} l\tau - 1}{l\tau}\right) - \frac{j}{\tau^2}\left(\frac{l}{2} - s\right);$$

$$q(s) = \alpha\tau \operatorname{sh} \tau s + \beta\tau \operatorname{ch} \tau s + \frac{j}{\tau^2};$$

$$u(s) = -\alpha\tau^2 \operatorname{ch} \tau s - \beta\tau^2 \operatorname{sh} \tau s.$$

Further solution of the problem is carried out according to the method described above.

Similarly to (25), we find maximal bucklings in the sections placed far from the free end

$$x_{\max} = x(l/2) = -\frac{jl^4}{384}\cdot\frac{3}{(l\tau/4)^2}\left(1 - \frac{\operatorname{th}(l\tau/4)}{l\tau/4}\right).$$

Decomposition of the expression containing $\xi = l\tau/4$ in a series gives

$$\frac{3}{\xi^2}\left(1 - \frac{\operatorname{th}\xi}{\xi}\right) = 1 - \frac{2}{5}\xi^2 + \frac{17}{105}\xi^4 - \frac{62}{945}\xi^6 + \ldots.$$

With $t = 0$, the row equals 1, and the buckling coincides with (10). As the parameter ξ increases, the expression in parentheses heads to 1, and the multiplier $3/\xi^2$ decreases. This confirms that the increase in the axial force of tension t makes the buckling of the rod between the supports decrease.

Conclusions. When a long casing string on the centralizers is pushed into a horizontal well, it experiences transverse and longitudinal bends under its own weight and frictional forces causing axial compression forces. The latter represent variables in length and depend on support reactions which, however, can be found in the first approximation in non-friction cases.

Geometric and power parameters of pipe deformations on the sections between the supports can be determined from the system of equations of the compatibility of section turns and the support point equilibrium obtained by solving the differential equation of the longitudinal-porous bend of the elastic rod. These equations contain axial force parameters, which are also derived from the proposed system of algebraic equations.

The solution of the problem becomes possible since the system of equations accounts for transverse forces obtained on the basis of the established relation between transverse and axial forces in the column and reactions and friction on the supports. Due to the value of axial forces, calculated in the first approximation, the system becomes linear, and its iterative solution allows us to find the desired parameters with high accuracy.

Our method takes into account additional moments of frictional forces affecting centralizers and the influence of wells deviations from the horizontal on the change of deformation-force parameters. The offered formulas help to calculate the optimal separation of the casing centralizers and the solution of the problem is provided for the case of its reverse motion.

The results obtained might serve for the analysis of the stress-deformed state of the casing column in the technological process of a horizontal well construction which allows oilmen to increase its reliability and durability.

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Деформування довгої обсадної колони на центраторах при встановленні в горизонтальну свердловину

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Мета. Розроблення методики розрахунку параметрів напружено-деформованого стану стисненої обсадної колони, яку проштовхують у горизонтальну свердловину під час її спорудження.

Методика. Задача розв'язана шляхом інтегрування диференціального рівняння поздовжнього згину довгого стрижня під дією власної ваги. У першому наближенні знайдені реакції на опорах-центраторах без урахування осьових сил тертя. Для визначення деформаційних і силових параметрів на ділянках між опорами застосована система алгебраїчних рівнянь на основі рівнянь сумісності деформацій і рівноваги згинальних моментів в опорних перетинах.

Результати. Знайдено загальний розв'язок основного диференціального рівняння деформацій горизонтальної колони труб з урахуванням тертя та осьових сил, що діють під час її просування. За його допомогою виведені формули для розрахунку прогинів, кутів поворотів перетинів стрижня, його внутрішніх згинальних моментів і поперечних сил на ділянках між опорами. Враховані додаткові моменти сил тертя, що діють на центраторах. Знайдені розв'язки задачі для випадку протилежного напрямку руху обсадної колони.

Наукова новизна. Запропоноване рівняння зв'язку між поперечними й поздовжніми силами в довгому стрижні та реакціями й силами тертя на опорах. Система рівнянь доповнена рівняннями поперечних сил, що дало змогу одночасно визначати осьові стискальні сили. Розроблена методика лінеаризації системи алгебраїчних рівнянь та її ітераційного розв'язання з високою точністю.

Практична значимість. Отримані результати спрямовані на врахування вимог технології спорудження горизонтальної свердловини. Виведені формули для розрахунку оптимальної відстані між центраторами. Розглянуто вплив відхилень напрямку ділянок свердловини від горизонталі на зміну напружено-деформованого стану обсадної колони, що дозволяє підвищити надійність і довговічність її експлуатації.

Ключові слова: *горизонтальна свердловина, обсадна колона, пружний стрижень, поздовжній згин, сила тертя, осьове стискання*

Деформирование длинной обсадной колонны на центраторах при установке в горизонтальную скважину

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Цель. Разработка методики расчета параметров напряженно-деформированного состояния сжатой обсадной колонны, которую проталкивают в горизонтальную скважину при ее сооружении.

Методика. Задача решена путем интегрирования дифференциального уравнения продольного изгиба длинного стержня под действием собственного веса. В первом приближении найдены реакции на опорах-центраторах без учета осевых сил трения. Для определения деформационных и силовых параметров на участках между опорами применена система алгебраических уравнений на основе уравнений совместности деформаций и равновесия изгибающих моментов в опорных сечениях.

Результаты. Найдено общее решение основного дифференциального уравнения деформаций горизонтальной колонны труб с учетом трения и осевых сил, действующих во время ее продвижения. С его помощью выведены формулы для расчета прогибов, углов поворотов сечений стержня, его внутренних изгибающих моментов и поперечных сил на участках между опорами. Учтены дополнительные моменты сил трения, действующие на центраторы. Найдено решение задачи для случая противоположного направления движения обсадной колонны.

Научная новизна. Предложено уравнение связи между поперечными и продольными силами в длинном стержне и реакциями и силами трения на опорах. Система уравнений дополнена уравнениями поперечных сил, что позволило одновременно определять осевые сжимающие силы. Разработана методика линеаризации системы алгебраических уравнений и ее итерационного решения с высокой точностью.

Практическая значимость. Полученные результаты направлены на учет требований технологии сооружения горизонтальной скважины. Выведены формулы для расчета оптимального расстояния между центраторами. Рассмотрено влияние отклонений направления участков скважины от горизонтали на изменение напряженно-деформированного состояния обсадной колонны, что позволяет повысить надежность и долговечность ее эксплуатации.

Ключевые слова: *горизонтальная скважина, обсадная колонна, упругий стержень, продольный изгиб, сила трения, осевое сжатие*

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