UDC 62-83

DOI: 10.29202/nvngu/2019-3/11

V. O. Volkov, Cand. Sc. (Tech.), Assoc. Prof.,
orcid.org/0000-0003-1262-3988

Dnipro University of Technology, Dnipro, Ukraine, e-mail: green_stone@ukr.net

OPTIMIZATION OF GENERAL LOSSES OF THE ENERGY OF THE FREQUENCY-REGULATED PUMPING AGGREGATE FOR START-BRAKING REGIMES

Purpose. Analytical calculation and optimization of the total energy losses for a frequency-regulated centrifugal pumping aggregate during start-braking regimes.

Methodology. Variational calculi, mathematical interpolation and computer simulation were used.

Findings. Analytical dependencies are obtained that allow calculating and optimizing the total energy losses of the frequency-regulated centrifugal pump set in the start-braking regimes. The quasi-optimal form of tachograms and the optimal values of the acceleration and deceleration time of this aggregate are defined, at which the minimization of its total energy losses in the start-braking regimes is ensured. Solutions for calculating energy consumption, as well as hydraulic, electromechanical and energy processes for the centrifugal pumping aggregate are implemented.

Originality. For the first time, analytical dependencies were obtained for calculating the total energy losses of a frequency-regulated pump aggregate in the start-braking regimes. A "U"-shape type of the dependences of the total energy losses of a given aggregate on the duration of its acceleration and deceleration time for different velocity trajectories is established. A quasi-optimal trajectory of velocity variation is proposed in the form of a temporal function of the hyperbolic sine with a variable value of the coefficient in its argument, at which the total losses of the pump aggregate in the start-braking regimes are minimal. A comparison is made of the total energy losses of the pump set for a quasi-optimal velocity trajectory with known other trajectories for different duration time of the start-braking regimes, which made it possible to estimate the energy saving achieved thereby.

Practical value. Introduction of the obtained results allows reducing to the minimum possible values the unproductive energy losses for the centrifugal pump aggregate in the start-braking regimes.

Keywords: *frequency-regulated pumping aggregate, total energy losses, start-braking regimes*

Introduction. Taking into account the widespread introduction of speed-controlled pumping aggregates (created primarily on the basis of frequency-controlled asynchronous short-circuited engines) in the housing and utilities sector and industry in Ukraine and other countries of the world, as well as their operation often in intensive starting and braking regimes (for example, sewage pumping stations with the number of inclusions/shutdowns up to one hundred per day), it becomes relevant and practical to quantify and minimize total energy loss for these aggregates in the start-braking regimes.

Literature review. From the analysis of domestic and foreign scientific and technical literature it has been established that the overwhelming part of the well-known publications devoted to the calculation and study of energy regimes and characteristics of pumping installations, is limited to consideration of steady-state regimes only: in [1, 2] – the consumption of active power is estimated when regulating the speed of a centrifugal pump (CP); in [2] – the forward and frequency starts of the pump engine are additionally investigated and compared; in [3] – power losses in a frequency-regulated CP are investigated taking into account typical graphs of changes in its load; [4] analyzes the total power consumption by parallel operating centrifugal pumping aggregates with frequency regulated asynchronous engines (FRAE), determines the values of the coefficient of efficiency of the cylinder and the engine when changing the fluid flow and pump speed, suggests energy saving management of these pumping installations; in [5] – a strategy for controlling several parallel-speed-controlled pumping aggregates operating in parallel was developed and investigated, ensuring an optimal load-sharing between the pumps, which minimizes the overall energy consumption of the pumps; in [6] – the minimization of energy consumption by frequency-controlled pumping aggregates, achieved by maintaining the optimum pressure value in these systems, was considered as applied to land-reclamation irrigation systems; in [7], for a frequency-controlled pump aggregate used in the water supply system of a multi-storey building, the assessment of its total active power loss and efficiency when changing speed and load was made.

Only completely occasional publications in the well-known scientific and technical literature are devoted to the research and minimization of electrical (in the article by Arribas J. R., Vega Gonzales C. M. Vector Control of Pumping and Ventilation Induction Motor Drives) or electromagnetic [8, 9] energy losses of FRAE in start-braking regimes. At the same time, in [8] – the minimization of the main electromagnetic energy losses of the FRAE loaded with PC is carried out by setting the optimal type of trajectory of the pump speed change in the start-braking regimes, and in [9] minimizing the specified energy losses the definitions of the proposed analytical dependencies and the next task of the optimal durations of acceleration and deceleration times of the FRAE loaded with PC are performed. In all of the three recent publications listed above, the power losses of this pump are not taken into account when varying its speed in these regimes. Since, in practice, the power losses of the pump usually exceed the power losses in the engine, it is not possible to implement energy-efficient control of them during the start-braking regimes, without taking into account the energy losses in the pump aggregate.

Purpose. Analytical calculation and optimization of the total energy loss for a frequency-regulated centrifugal pumping aggregate under start-braking regimes.

Results. The following assumptions were made:

- the start-braking regimes of the frequency regulated pump aggregate (PA) were considered with a fully open throttle valve applied to a hydraulic network characterized by a non-zero value of its static head (back pressure) and a non-return valve in the outlet of the pump (which is equal to or less than static head of the network – closed, and for values of pump head exceeding the static head of the network, – open);

- back valve was taken without inertia;

- when calculating the hydraulic parameters of the pump and the hydraulic network, a relative system of units was used (in which the nominal values of the output head and the flow rate of the test pump were taken as the base values of the head and the flow);

- for the calculation of electromechanical and energy processes and parameters of the induction engine driving the pump, the relative system of units common for asynchronous engines was used [9];

- the automatic control system (ACS) supports the most common frequency control law for vector control: with the constancy of the engine rotor flux linkage module Ψ_r equal to its nominal value [9]; the choice of the vector control method of the FRAE is due to the need to ensure fast-acting acceleration and deceleration regimes with a normalized view of the trajectories of its speed change (which, as is known, is not achieved with the scalar control of the FRAE);

- in the total losses of power and energy of the frequency-controlled PA, only the main component of these losses was taken into account for the engine, caused by the main harmonic components of its phase currents and voltages [9];

- the following regimes were researched: acceleration of PA from zero to nominal ω_n speed and deceleration from nominal speed ω_n to zero speed (where in calculations the value ω_n in relative units was assumed to be equal to a unit: $\omega_n = 1$ p. u., which corresponds in relative units to the nominal synchronous speed of the asynchronous engine [9]);

- research studies were carried out for linear, parabolic and energy-saving quasi-optimal trajectories of the speed of PA from [9] (with the parameters of an asynchronous engine of type АО3-315-4У3 with power of 200 kW and CP type SE450/95-2a presented in [9]), operating in the regime of pumping waste water at the sewage pumping station.

At the first stage, we present and analyze the analytical dependencies from the book [10] for calculating (in relative units) the hydraulic and electromechanical parameters of the speed-regulated CP.

In the range of low values ω of pump speeds (no more boundary $\omega_{\lim}$) pump head H generated is not higher than the static head H_{st} of the hydraulic network: $H \le H_{st}$. In this case, due to the closed state of the check valve, there is no flow ($Q = 0$) of the pump, and the value of the head created by the pump is calculated from the relationship

$$H = H_{on} \cdot \omega^2, \quad (1)$$

where H_{on} is the pump head value corresponding to the nominal pump speed at zero flow ($Q = 0$) and determined from the passport characteristics of a particular pump. According to (1), the value of the boundary velocity is defined as

$$\omega_{\lim} = \sqrt{H_{st}/H_{on}}.$$

In the considered range of low velocities, the CP operates (taking into account the closed valve) at idle (without backpressure), and its static moment is calculated from the dependence

$$M_s = M_r + (M_o - M_r) \cdot \omega^2, \quad (2)$$

where $M_r = (0.05 - 0.1)M_{sn}$ is the static moment of the pump running; M_o is the value of the static torque of CP, corresponding to its work at idle (at $Q = 0$) with a nominal speed; M_{sn} is the nominal value of the static moment of the CP, corresponding to its functioning at the nominal speed at the nominal values of its head and feed. The value of the torque M_o can be for CP (depending on the coefficient of rapidity, according to the book by Florinskiy M. M., Rychagov V. V. Pumps and pump stations) from 0.4 to 0.7 of the nominal value M_{sn}. For the considered pump SE450/95-2a, the coefficient of rapidity n_S, is calculated in this book from the formulas

$$n_s = n_n \cdot \sqrt{N_n} \Big/ \left(H_n^4 \cdot \sqrt{H_n} \right); \quad N_n = \left(1000/75 \right) \cdot Q_n H_n,$$

equal to 69.6 rpm (which corresponds to the same book value $M_o = 0.4M_{sn}$), where $n_n = 1500$ rpm, $H_n = 78$ m, $Q_n = (1/9)$ m^3/s.

In the range of pump speeds, ω is more boundary: $\omega_{\lim} < \omega \le \omega_n$ the pressure created by the pump exceeds the static head H_{st} of the hydraulic network ($H > H_{st}$), as a result, the back valve goes into the open state, the pump creates flow Q and head H, whose values for a particular speed-regulated pump are interconnected by its $H - Q$-characteristics of the form [7]

$$H = H_{on} \cdot \omega^2 + A \cdot \omega \cdot Q - B \cdot Q^2, \quad (3)$$

(given in the form of graphical dependencies for pumps by their manufacturers at $\omega = 1$). The values of constant coefficients A and B in expression (3) are determined from the relations [7]

$$\left.\begin{aligned} A &= \frac{\left(H_{on} - H_n\right) Q_1^2 - \left(H_{on} - H_1\right) Q_n^2}{Q_1 \cdot Q_n \cdot \left(Q_n - Q_1\right)} \\ B &= \frac{\left(H_{on} - H_n\right) Q_1 - \left(H_{on} - H_1\right) Q_n}{Q_1 \cdot Q_n \cdot \left(Q_n - Q_1\right)} \end{aligned}\right\},$$

based on three (for example, two extreme and one middle) points on the working section $H - Q$-characteristics of the pump: 1) $Q = 0$, $H = H_{on}$; 2) $Q \approx 0.5Q_n$, $H = H_1$; 3) $Q = Q_H$, $H = H_n$ — corresponding to the operation of the pump at the nominal speed (at $\omega = 1$), where Q_n and H_n are the nominal values (in relative units), respectively, of the pump flow and head. For the investigated CP, these values are equal: H_{on} = 1.4 p.u., A = –0.38 p.u.; B = 0.02 p.u. To create the maximum value of the efficiency of the speed-regulated ω CP, correspond dependencies (3) for its pump flow and head

$$\left.\begin{aligned} H &= H_{on}\omega^2 + A\omega\sqrt{\frac{\omega^2 - \left(H_{st}/H_{on}\right)}{1 - \left(H_{st}/H_{on}\right)}} - \\ &- B\left[\frac{\omega^2 - \left(H_{st}/H_{on}\right)}{1 - \left(H_{st}/H_{on}\right)}\right], \; Q = \sqrt{\frac{\omega^2 - \left(H_{st}/H_{on}\right)}{1 - \left(H_{st}/H_{on}\right)}} \end{aligned}\right\}. \quad (4)$$

In the considered working range of change of speed: $\omega_{\lim} < \omega \le \omega_n$ instantaneous values of pump efficiency $\eta_{??}$ are calculated in the form

$$\eta_{cp} = 1 - (1 - \eta_{cp.n})/\omega^{3.6}, \quad (5)$$

through the nominal value $\eta_{cp.n}$ of this efficiency (equal to 58.5 % for the investigated CP and given in its passport data for the nominal values of the speed ω = 1 p.u. and feed $Q = Q_n$ [9]).

In this case, the instantaneous value of the static torque of the pump is determined from the dependency

$$M_s = \rho \cdot g \cdot (Q \cdot Q_b) \cdot (H \cdot H_b)/[M_b \cdot \eta_{cp} \cdot (\omega \cdot \omega_b)], \quad (6)$$

where Q_b [m^3/s], H_b [m] and ω_b [rad/s] are basis values, respectively, of the supply, head and speed of the CP considered (which are assumed to be equal to their respective nominal values); ρ = 1050 [kg/m^3] is the specific gravity of the pumped liquid; g = 9.81 [m/s^2] is acceleration of free fall; M_b [Nm] is the basic value for the electromagnetic torque of the engine.

In the range of speeds $0 < \omega \le \omega_n$, corresponding to the start-braking regimes PA, the values of the electromagnetic torque and the module I_1 of the vector of the stator current of the FRAE are dependencies [9]

$$M = M_s + J \cdot \omega'; \quad I_1 = [(Y_{rn}/L_m)^2 + (M/k_r\Psi_{rn})^2]^{0.5},$$

where $\omega' = d\omega/dt$ is time derivative with respect to time; L_m and k_r, respectively, are the magnetizing inductance and coupling coefficient of the engine rotor.

At the second stage, we obtain the analytical dependences for calculating the total power and energy losses in the NR during the start-braking regimes.

Total power losses ΔP_{pa} PA represent the sum of the power loss in the centrifugal pump ΔP_{cp} and the total power loss ΔP_{en} of the engine

$$\Delta P_{pa} = \Delta P_{cp} + \Delta P_{en}. \quad (7)$$

At the same time, the total power loss for a speed-regulated CP is equal to the difference between the mechanical power consumed P_{meh} and the useful hydraulic power developed by it P_{cp}

$$\left.\begin{aligned} &\Delta P_{cp} = P_{meh} - P_{cp}; \quad P_{meh} = M_s \cdot \omega \\ &P_{cp} = \begin{cases} 0 - & \text{at } 0 < \omega \le \omega_{\lim} \\ \rho g\left(Q \cdot Q_b\right)\left(H \cdot H_b\right)/P_b - \text{at} & \omega_{\lim} < \omega \le \omega_n \end{cases} \end{aligned}\right\}, \quad (8)$$

where P_b is the base value for the power of the FRAE calculated from [9]. Moreover, the ratio for the mechanical power P_{meh} in (8) contains the value of the static torque M_s calculated from (2) or (6) for the corresponding current speed range ($0 < \omega \le \omega_{\lim}$ or $\omega_{\lim} < \omega \le \omega_n$) of the pump.

Substituting (6) into (8), we obtain, taking into account (5) for the range of pump speeds $\omega_{\lim} < \omega \le \omega_n$, dependence for the total power loss

$$\begin{aligned} \Delta P_{cp} &= \rho g\left(Q \cdot Q_b\right) \cdot \left(H \cdot H_b\right) \cdot \left(1 - \eta_{cp}\right)/\left(\eta_{cp} \cdot P_b\right) = \\ &= \frac{\rho g\left(Q \cdot Q_b\right) \cdot \left(H \cdot H_b\right)}{P_b} \cdot \left[\frac{1 - \eta_{cp.n}}{\omega^{0.36} + \eta_{cp.n} - 1}\right]. \end{aligned} \quad (9)$$

In this case, the nominal value of the loss of power of the CP is determined from the dependency

$$\Delta P_{cp.n} = \frac{\rho g\left(Q_n \cdot Q_b\right) \cdot \left(H_n \cdot H_b\right)}{P_b} \cdot \left(\frac{1}{\eta_{cp.n}} - 1\right), \quad (10)$$

where Q_n = 1 p.u., H_n = 1 p.u. are nominal values of flow and pump head, respectively.

After substituting (4) in the last expression from (9), we obtain, taking into account (2, 8 and 10), the dependencies for the ratio Δp of the instantaneous ΔP_{cp} and nominal $\Delta P_{cp.n}$ values of the power losses of the CP

$$\left.\begin{aligned} &\Delta p = \frac{\Delta P_{cp}}{\Delta P_{cp.n}} = \begin{cases} \Delta p_1 - \text{at } 0 < \omega \le \omega_{\lim} \\ \Delta p_2 - \text{at } \omega_{\lim} < \omega \le \omega_n \end{cases} \\ &\Delta p_1 = \left[M_r \cdot \omega + \left(M_o - M_r\right) \cdot \omega^3\right]/\Delta P_{cp} \\ &\Delta p_2 = QH\left(\frac{\eta_{cp.n}}{\omega^{0.36} + \eta_{cp.n} - 1}\right) = \\ &= \frac{\left[H_{on}\omega^2 \cdot x + A\omega \cdot x^2 - B \cdot x^{3/2}\right]\eta_{cp.n}}{\left(\omega^{0.36} + \eta_{cp.n} - 1\right)} \\ &x = \left[\frac{\omega^2 - \left(H_{st}/H_{on}\right)}{1 - \left(H_{st}/H_{on}\right)}\right]^{0.5} \end{aligned}\right\}. \quad (11)$$

Electromagnetic and mechanical power losses of the FRAE are calculated taking into account [9] in the form

$$\Delta P_{em} = a + b \cdot M^2 + c \cdot \omega^{1.3} \quad \text{and} \quad \Delta P_{meh} = d \cdot \omega^2, \quad (12)$$

where the coefficients a, b, c and d are found through the parameters of a particular engine from the ratios

$$\left.\begin{aligned} a &= \left(\Psi_r/L_m\right)^2 \cdot \left(R_s + 0.005P_n/\eta_{en.n}\right), \quad d = \Delta P_{meh.n} \\ b &= \left(R_s + k_r^2 R_r + 0.005P_n/\eta_{en.n}\right) \cdot \left(k_r^2 \Psi_r^2\right), \quad c = \Delta P_{st.n} \end{aligned}\right\}.$$

In the last ratios, the following notation is used: R_s and R_r are active resistances of the stator and rotor engine, respectively; P_n and $\eta_{en.n}$ are the nominal values, respectively, of the useful power on the shaft and the efficiency of the engine; $\Delta P_{st.n}$ and $\Delta P_{meh.n}$ are the nominal values, respectively, in steel loss and engine mechanical losses. Moreover, by means of the term $0.005P_n/\eta_{en.n}$, additional losses of engine power are taken into account in the composition of its electromagnetic losses [9].

Based on (7–9), the total energy losses (TEL) of the pumping unit during acceleration and deceleration are determined in the form, respectively

$$\Delta W_{pa} = \int_0^{t_a} \Delta P_{pa} \cdot dt \quad \text{and} \quad \Delta W_{pa} = \int_0^{t_d} \Delta P_{pa} \cdot dt, \quad (13)$$

where t_a and t_d are the durations of acceleration and deceleration times, respectively; t is the current time, counted from the beginning and during the acceleration regimes ($0 \le t \le t_a$) or deceleration ($0 \le t \le t_d$).

At the third stage, using the methods of variational calculus (discussed in the book by Andreeva E. A., Tsirulyova V. M. Variational Calculus and Optimization Methods), we will optimize the TEL for PA in the start-braking regimes.

For this, in the range of speeds: $0 \le \omega \le \omega_n$ corresponding to the considered start-braking regimes PA (with reference to the parameters of the investigated CP when $H_{st}/H_n = 0.3$), from analytical expressions (2), (6, 11) static graphs torque $M_s(\omega)$ and the ratio $\Delta p(\omega)$ of total power losses to their nominal value are calculated and built in Fig. 1. Due to the cumbersome formulas describing these values in the speed range $\omega_{\lim} < \omega \le \omega_n$, these values are interpolated (with a deviation of not more than 0.8 %, as shown in Fig. 1 by circles) with simplified analytical dependencies of the form

$$\begin{aligned} M_s &\approx 1.104 \cdot \omega - 0.0448; \\ \Delta p &= (\omega + 0.045)^{2.4} - 0.11. \end{aligned} \quad (14)$$

With this in mind and based on relations (7, 11, 12), we obtain the dependencies for the calculation and total power loss PA during the first (by $0 \le \omega \le \omega_{\lim}$)

$$\begin{aligned} \Delta P_{pa1} &= \Delta P_{cp1} + \Delta P_{em1} + \Delta P_{meh} = \\ &= (0.08\omega + 0.184\omega^2) + \{a + b\,[M_r + (M_o - M_r)\omega^2 + \\ &+ J \cdot \omega']^2 + c \cdot \omega^{1.3} + d \cdot \omega^2\}, \end{aligned} \quad (15)$$

the second (with $\omega_{\lim} < \omega \le \omega_n$) speed ranges

$$\begin{aligned} \Delta P_{pa2} &= \Delta P_{cp2} + \Delta P_{em2} + \Delta P_{meh} = \\ &= \Delta P_{cp.n}[(\omega + 0.045)^{2.4} - 0.11] + \{(a + 0.201b) - \\ &- 0.989 \cdot b\omega + (1.219 \cdot b + d) \cdot \omega^2 + 2.208 \cdot bJ\omega \cdot \omega' - \\ &- 0.896 \cdot bJ\omega' + bJ^2(\omega')^2 + c \cdot \omega^{1.3}\}. \end{aligned} \quad (16)$$

According to the theory of variation calculus, for the function ΔP_{pa}, Euler's equation corresponds to the extremal value of the integrals of (13)

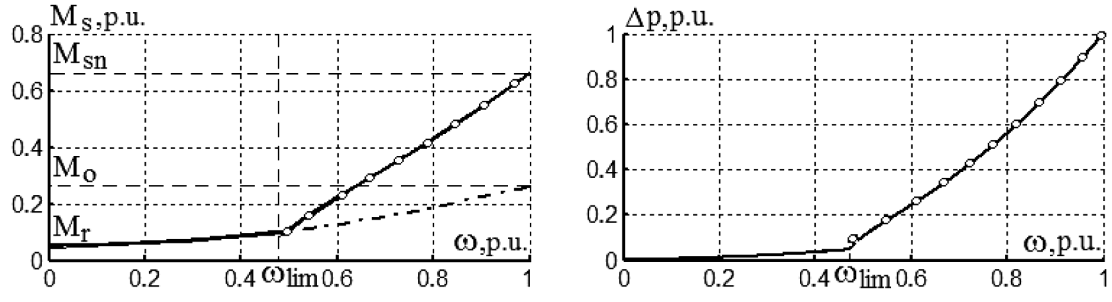
$$\begin{aligned} &\frac{\partial^2\left(\Delta P_{pa}\right)}{\partial\omega' \cdot \partial\omega'} \cdot \omega'' + \frac{\partial^2\left(\Delta P_{pa}\right)}{\partial\omega \cdot \partial\omega'} \cdot \omega' + \\ &+ \frac{\partial^2\left(\Delta P_{pa}\right)}{\partial\omega \cdot \partial t} - \frac{\partial\left(\Delta P_{pa}\right)}{\partial\omega} = 0. \end{aligned} \quad (17)$$

Substituting in (17) the dependences ΔP_{pa1} from (15) or ΔP_{pa2} from (16), we transform this equation for the first and second speed ranges to the form

$$\left.\begin{aligned} &\omega'' = y_1\omega + y_2\omega^{0.3} + y_3\omega^3 + y_4 \ \text{ at } \ 0 \le \omega \le \omega_{\lim} \\ &\omega'' = z_1\omega + z_2\omega^{0.3} + z_3\left(\omega + 0.045\right)^3 + z_4 \\ &\qquad\qquad \text{at } \ \omega_{\lim} \le \omega \le \omega_n \end{aligned}\right\}, \quad (18)$$

where the values of constant coefficients y_1, y_2, y_3, y_4 and z_1, z_2, z_3, z_4 are given in Table 1.

Due to the nonlinear form of the differential equations of (18), the solution of each of them is performed using the Runge-Kutta numerical method taking into account the boundary conditions during acceleration: $\omega(0) = 0$, $\omega(t_{a1}) = \omega_{\lim}$ – for the first and $\omega(0) = \omega_{\lim}$, $\omega(t_{a2}) = \omega_n$ – for the second speed ranges or when braking: $\omega(0) = \omega_n$, $\omega(t_{d2}) = \omega_{\lim}$ – for the second and



a *b*

Fig. 1. Interpolation (when $\omega_{\lim} < \omega \le \omega_n$ for $H_{st}/H_n = 0.3$) dependencies:
a – for $M_s(\omega)$; b – $\Delta p(\omega)$ centrifugal pump (solid line – the original form, circles – interpolated)

Table 1

The values of the coefficients for the first and second equations of (18)

Speed ranges	$0 \le \omega \le \omega_{\text{lim}}$					$\omega_{\text{lim}} < \omega \le \omega_n$				
Values	y_1	y_2	y_3	y_4	K_1	z_1	z_2	z_3	z_4	K_2
Value 10^{-5}, o. e.	19.94	1.170	0.387	4.238	41.53	5.580	1.170	34.78	-2.120	41.53

$\omega(0) = \omega_{\text{lim}}$, $\omega(t_{d1}) = 0$ – for the first speed ranges (where t_{a1}, t_{d1} and t_{a2}, t_{d2} are the durations of times for the specified first and second speed ranges). For the considered pumping aggregate, calculated at $\omega_{\text{lim}} = 0.4629$ p. u. and $t_a = t_d = 1$ p. u., the optimal types of trajectories of speed change in the first and second speed ranges are shown in graphs 5 in Fig. 2, *a, b.* It is established that the best approximation (with a relative standard deviation of less than 0.9 %) to these graphs are mathematical dependencies during acceleration

$$\left.\begin{aligned} &\omega = \omega_{\text{lim}} \cdot \frac{\text{sh}\left(\xi^* \cdot \sqrt{K_1} \cdot t\right)}{\text{sh}\left(\xi^* \cdot \sqrt{K_1} \cdot t_{a1}\right)} \quad \text{at} \quad 0 \le t \le t_{a1} \\ &\omega = \omega_{\text{lim}} + \left(\omega_n - \omega_{\text{lim}}\right) \cdot \frac{\text{sh}\left(\xi^* \sqrt{K_2} \cdot t\right)}{\text{sh}\left(\xi^* \sqrt{K_2} \cdot t_{a2}\right)} \\ &\qquad\qquad \text{at} \quad 0 \le t \le t_{a2} \end{aligned}\right\}, \quad (19)$$

and deceleration

$$\left.\begin{aligned} &\omega = \omega_{\text{lim}} + \left(\omega_n - \omega_{\text{lim}}\right) \times \\ &\quad \times \frac{\text{sh}\left[\xi^* \cdot \sqrt{K_2} \cdot \left(t_{d2} - t\right)\right]}{\text{sh}\left(\xi^* \cdot \sqrt{K_2} \cdot t_{d2}\right)} \quad \text{at} \quad 0 \le t \le t_{d2} \\ &\omega = \omega_{\text{lim}} \cdot \frac{\text{sh}\left[\xi^* \sqrt{K_1} \cdot \left(t_{d1} - t\right)\right]}{\text{sh}\left(\xi^* \sqrt{K_1} \cdot t_{d1}\right)} \quad \text{at} \quad 0 \le t \le t_{d1} \end{aligned}\right\}, \quad (20)$$

where the coefficients K_1 and K_2 are equal

$$K_1 = y_1 + y_2 + y_3 \quad \text{and} \quad K_2 = z_1 + z_2 + z_3. \quad (21)$$

At the same time, the dependences from (19, 20) and the corresponding graphs in Fig. 2 (shown in this and subsequent figures as numeral 1) are called "quasi-optimal" tachograms of frequency-regulated PA (which provide close to minimal values of TEL for PA in the start-braking regimes). The correction coefficients ξ_1^*, ξ_2^* calculated for quasi-optimal tachograms are shown as graphical dependences in Fig. 3 (where the index of these coefficients corresponds to the number of the considered speed range).

At the fourth stage, for the parameters of the explosive PA , its TEL values were calculated depending on the duration of the time t_{a1}, t_{a2} and t_{d1}, t_{d2} acceleration and deceleration, respectively, for the first second velocity ranges. These calculations are performed from (19, 20) for quasi-optimal trajectories of speed, and also for comparison, for linear

$$\left.\begin{aligned} &\omega = \omega_{\text{lim}} \cdot \left(t/t_{a1}\right) && \text{at} \quad 0 \le t \le t_{a1} \\ &\omega = \omega_{\text{lim}} + \left(\omega_n - \omega_{\text{lim}}\right) \cdot \left(t/t_{a2}\right) && \text{at} \quad 0 \le t \le t_{a2} \\ &\omega = \omega_{\text{lim}} + \left(\omega_n - \omega_{\text{lim}}\right) \cdot \left(t_{d2} - t\right)/t_{d2} && \text{at} \quad 0 \le t \le t_{d2} \\ &\omega = \omega_{\text{lim}} \cdot \left(t_{d1} - t\right)/t_{d1} && \text{at} \quad 0 \le t \le t_{d1} \end{aligned}\right\}, \quad (22)$$

parabolic concave shape

$$\left.\begin{aligned} &\omega = \omega_{\text{lim}} \cdot \left(t/t_{a1}\right)^2 && \text{at} \quad 0 \le t \le t_{a1} \\ &\omega = \omega_{\text{lim}} + \left(\omega_n - \omega_{\text{lim}}\right) \cdot \left(t/t_{a2}\right)^2 && \text{at} \quad 0 \le t \le t_{a2} \\ &\omega = \omega_{\text{lim}} + \left(\omega_n - \omega_{\text{lim}}\right) \cdot \left(\frac{t_{d2} - t}{t_{d2}}\right)^2 && \text{at} \quad 0 \le t \le t_{d2} \\ &\omega = \omega_{\text{lim}} \cdot \left[\left(t_{d1} - t\right)/t_{d1}\right]^2 && \text{at} \quad 0 \le t \le t_{d1} \end{aligned}\right\}, \quad (23)$$

and parabolic convex shape of tachograms

$$\left.\begin{aligned} &\omega = \omega_{\text{lim}} \cdot \left\{1 - \left[\left(t_{a1} - t\right)/t_{a1}\right]^2\right\} && \text{at} \quad 0 \le t \le t_{a1} \\ &\omega = \omega_{\text{lim}} + \left(\omega_n - \omega_{\text{lim}}\right) \times \\ &\quad \times \left\{1 - \left[\left(t_{a2} - t\right)/t_{a2}\right]^2\right\} && \text{at} \quad 0 \le t \le t_{a2} \\ &\omega = \omega_{\text{lim}} + \left(\omega_n - \omega_{\text{lim}}\right)\left[1 - \left(t/t_{d2}\right)^2\right] && \text{at} \quad 0 \le t \le t_{d2} \\ &\omega = \omega_{\text{lim}} \cdot \left[1 - \left(t/t_{d1}\right)^2\right] && \text{at} \quad 0 \le t \le t_{d1} \end{aligned}\right\}. \quad (24)$$

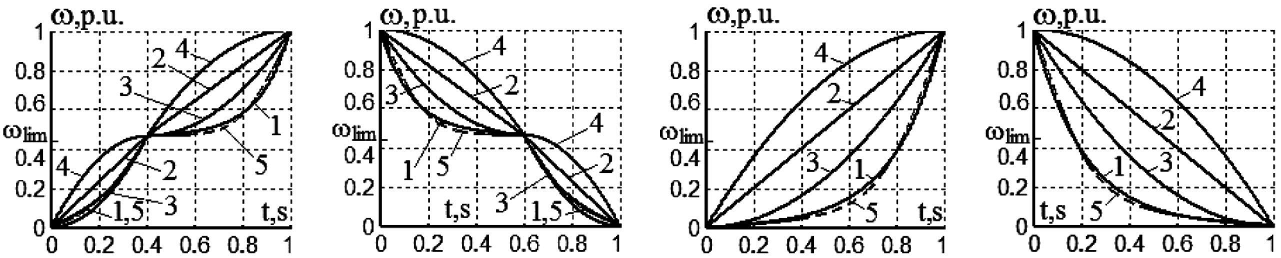
The graphs shown in all figures, referring to the linear, parabolic concave shape and parabolic convex shape to the tachograms, are designated by numbers 2, 3 and 4, respectively.

The results of these calculations of the TEL $\Delta W_{pa}(t_{a1})$, $\Delta W_{pa}(t_{a2})$ and $\Delta W_{pa}(t_{d1})$, $\Delta W_{pa}(t_{d2})$ corresponding to the acceleration and deceleration throughout the first and second speed ranges, are shown in the form of graphs in Fig. 4. The results of the calculation of the total (for both speed ranges) TEL

$$\Delta W_{pa}(t_a) = \Delta W_{pa1}(t_{a1}) + \Delta W_{pa2}(t_{a2});$$
$$\Delta W_{pa}(t_d) = \Delta W_{pa1}(t_{d1}) + \Delta W_{pa2}(t_{d2}),$$

are depicted as graphs in Fig. 5, *a* on these graphs: for the first speed range, the duration of acceleration and deceleration time was set as optimal (providing, according to Fig. 4, the minimum values of TEL for the corresponding types of PA tachograms or could be calculated from [9]) and for the second speed range – the durations of acceleration and deceleration time varied.

Additionally, in Fig. 5, *b* the graphs of the TEL are depicted and applied to the smooth type of tacho-



a *b* *c* *d*

Fig. 2. Types of the researched broken (a, b) and smooth (c, d) PA tachograms (1 – quasi-optimal; 2 – linear; 3, 4 – parabolic concave and convex shapes; 5 – optimal) during acceleration (a, c) and deceleration (b, d)

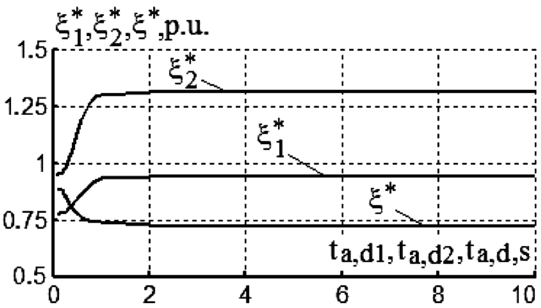


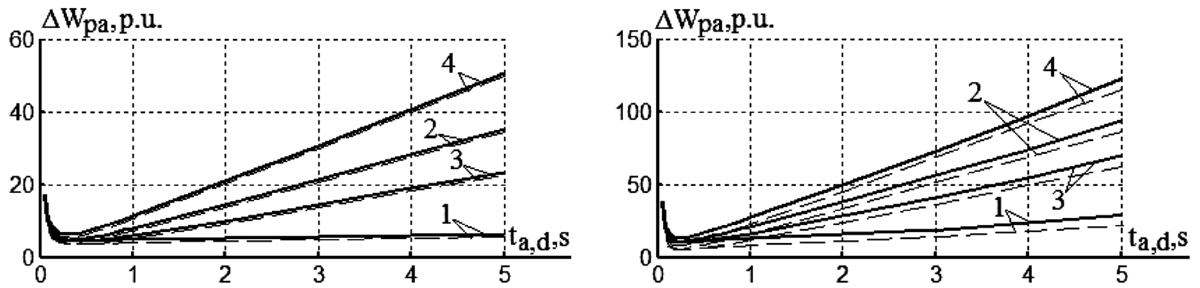
Fig. 3. Dependencies $\xi_1^*\left(t_{a,d1}\right)$*,* $\xi_2^*\left(t_{a,d2}\right)$ *and* $\xi^*\left(t_{a,d}\right)$

grams shown in Fig. 2, *c, d* with corresponding quasi-optimal, linear, parabolic concave and convex trajectories of the PA rate calculated from: the first dependencies in (19, 20), the first and third dependencies in (22, 23, 24) when replacing in them the parameters $\omega_{\lim}$, t_{a1}, t_{d1} respectively, by ω_n, t_a, t_d. At the same time, Fig. 3 shows calculated values (with varying durations $t_{a,d}$ of acceleration and deceleration times) of the correction factor ξ^* for a quasi-optimal smooth type of tachograms, corresponding to minimizing the value ΔW_{pa} of TEL for PA in acceleration and deceleration regimes.

Let us calculate the specific value k_Q of the volume of the pumped-over liquid through the PA to the TEL consumed in it during the start-braking regimes

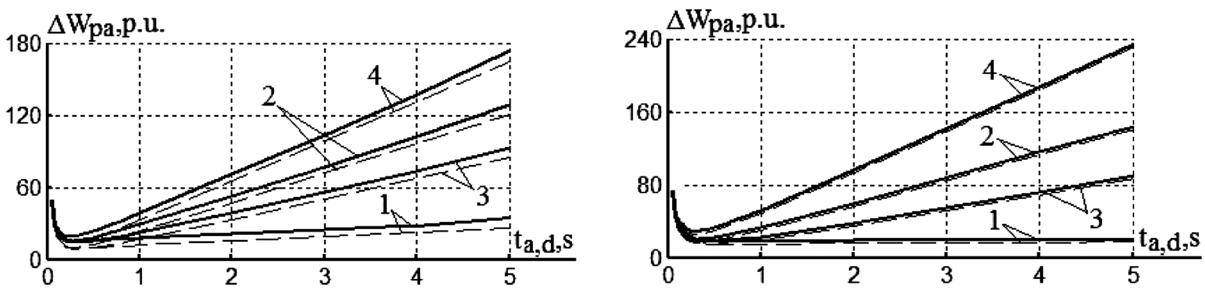
$$k_Q = \frac{\Delta P_{pa.n} \cdot \int_0^{t_{a,d}^o} Q \cdot dt}{Q_n \cdot \Delta W_{pa}^o};$$

$$\left.\begin{aligned} \Delta P_{na.n} &= \Delta P_{cp.n} + \Delta P_{en.n} \\ \Delta P_{en.n} &= P_{en.n}\left(1/\eta_{en.n} - 1\right) \end{aligned}\right\},$$



a *b*

Fig. 4. Dependences of the TEL $\Delta W_{pa}(t_{a,d1})$ *and* $\Delta W_{pa}(t_{a,d2})$ *at* $H_{st}/H_n = 0.3$ *of the speed ranges:*
a – for $0 \le \omega \le \omega_{\lim}$*; b – for* $\omega_{\lim} \le \omega \le \omega_n$ *different tachograms (1 – quasi-optimal; 2 – linear; 3, 4 – parabolic concave and convex shape, shown during acceleration by a solid line, when braking – dotted line)*



a *b*

Fig. 5. Dependences of TEL $\Delta W_{pa}(t_{a,d})$ *at* $H_{st}/H_n = 0.3$ *for broken (a) and smooth (b) types of tachograms:*
1 – quasi-optimal; 2 – linear; 3, 4 – parabolic concave and convex shape (shown at start by a solid line, when braking – by a dotted line)

where $\Delta P_{pa.n}$ and $\Delta P_{en.n}$ are the nominal values of the TEL, respectively, for the PA and the engine; $P_{en.n}$ and $\eta_{en.n}$ are the nominal values, respectively, of the power and efficiency of the engine. And, we note, for the established nominal regime PA: $k_Q = 1$.

The numerical values of the optimal (minimum) TEL ΔW_{pa}^o for PA in the start-braking regimes and the corresponding optimum acceleration t_a^o, t_{a1}^o and deceleration t_d^o, t_{d1}^o time, as well as the specific value, are presented in Table 2.

At the fifth stage, calculations are made of the previously presented analytical dependencies: hydraulic $Q(t)$, $H(t)$ and electromechanical $\omega(t)$, $M(t)$, $M_s(t)$, $I_1(t)$ and transient processes, as well as energy transients processes $\Delta P_{pa}(t)$, $\Delta P_{cp}(t)$, $\Delta P_{en}(t)$ and the current efficiency $\eta_{cp}(t)$ of the pump. To translate the values of these quantities from relative units into absolute ones, the first ones should be multiplied by the basic values for these quantities (which are given in Table 3).

Conclusions.

1. The obtained analytical dependences (11) for instantaneous power losses of a variable-speed regulated CP and their mathematical interpolation proposed in (14) (in the more limited speed range of speeds) makes it possible to carry out subsequent optimization of the total energy loss by using the methods of variation calculus PA in the start-braking regimes.

2. From Figs. 4, 5, it follows that for all values of the durations of acceleration and deceleration time, the largest values of the total energy loss PA are characteristic of the parabolic convex shape of the speed trajectory (therefore, its use is inappropriate) and the smallest – trajectories in the form of a hyperbolic sine with a variable value (according to Fig. 3) of correction factors ξ_1^*, ξ_2^*, ξ^*, in the argument (this trajectory is called "quasi-optimal").

3. It has been established that in both considered speed ranges (lower and more boundary speeds), the dependences of TEL $\Delta W_{pa1}(t_{a1})$, $\Delta W_{pa2}(t_{a2})$ and $\Delta W_{pa1}(t_{d1})$, $\Delta W_{pa2}(t_{d2})$ for PA, have a "U"-shaped type characterized by certain (optimal) values of acceleration t_{a1}^o, t_{a2}^o and deceleration t_{d1}^o, t_{d2}^o time, these losses are minimal for any kind of considered trajectories of speed (quasi-optimal, linear, parabolic concave or convex shape).

4. The total values of optimal time of t_a^o acceleration and t_d^o deceleration are in the form of the sum of the corresponding optimal times for the first and second speed ranges: $t_a^o = t_{a1}^o + t_{a2}^o$ and $t_d^o = t_{d1}^o + t_{d2}^o$. For the same types of tachograms (according to Figs. 4, 5 and Table 2), equality is established between themselves of the optimal acceleration and deceleration time: $t_a^o = t_d^o$, $t_{a1}^o = t_{d1}^o$, $t_{a2}^o = t_{d2}^o$. According to Fig. 5, with the same types of tachograms and identical values of the acceleration and deceleration times, the corresponding TEL values for PA are close to each other.

5. According to Table 2, it follows that with optimal durations of acceleration t_a^o and deceleration t_d^o time and a broken form of the trajectories of speed PA in actuating regimes, due to the transition from a linear, parabolic concave or convex tachograms to a quasi-optimal trajectory of speed, the total (in one cycle: "acceleration – deceleration") EPR in these regimes, respectively, by 5; 2 and 41 %. With the deviation of the lengths of the acceleration t_a and deceleration t_d time from their optimal values, the achieved decrease in total TEL when moving from a linear, parabolic concave or convex tachograms shape to a quasi-optimal trajectory of speed increases (according to Fig. 5, *a* with $t_a = t_d = 5$ s respectively by 3.8; 2.6 and 5.1 times).

6. It was revealed that the use of the optimal broken (according to Fig. 2, *a, b*) of the speed trajectories in the starting-brake regimes instead of the optimal smooth appearance of these trajectories (according to Fig. 2, *c, d*) allows from Table 3, reducing the optimal values of the TEL for centrifugal PA in the start-braking regimes by (20–35) %.

Table 2

Optimum values of total energy losses and durations of acceleration and deceleration times, PA

Regime	Acceleration						Deceleration					
Tachogram	smooth		broken				smooth		broken			
Magnitude	ΔW_{pa}^o	t_a^o	ΔW_{pa}^o	$t_{a(1)}^o$	t_a^o	k_Q	ΔW_{pa}^o	t_d^o	ΔW_{pa}^o	$t_{d(1)}^o$	t_d^o	k_Q
Dimension	p.u.	s	p.u.	s	s	p.u.	p.u.	s	p.u.	s	s	p.u.
quasi-optimal	17.30	0.76	14.29	0.39	0.64	0.973	11.74	0.76	8.737	0.39	0.64	1.60
linear	20.42	0.34	14.84	0.31	0.53	0.929	14.87	0.34	9.283	0.31	0.53	1.49
parab.(conc.)	18.21	0.52	14.63	0.43	0.67	0.777	12.65	0.52	8.881	0.43	0.73	1.28
parab.(conv.)	30.14	0.30	19.05	0.29	0.51	0.874	24.59	0.30	13.49	0.29	0.51	1.23

Table 3

Basic values

Magnitude	H	Q	ω	M, M_s	P, ΔP	ΔW	Ψ	I_1	R_s, R_r	J	t
Dimension	m	m^3/s	rad/s	N·m	kW	J	Wb	A	Om	kg·m^2	s
Value	78	1/9	157.1	1473	231.3	737	1.71	287	1.873	0.0299	0.01/π

7. It has been established that when optimizing the total energy losses of the PA in the start-braking regimes, the values of the optimal acceleration t_a^o and deceleration t_d^o time is smaller compared to the optimal time from [9], which minimizes the main electromagnetic energy losses FRAE. With this in mind, it is necessary to check the obtained values of the optimal time, based on the hydraulic impact created during them. If the optimum time corresponds to an unacceptable amount of water hammer, then the braking time should be increased to such values at which the created value of the water hammer becomes acceptable.

8. From the analysis of the value k_Q in Table 2, it was found that in the start-braking regimes with optimal acceleration and deceleration time, the best (largest) ratio between the volume of the PA liquid to its value TEL is characteristic of quasi-optimal and linear forms of tachograms, whose advantage also includes, according to Fig. 6, the smallest maximum values of the stator current of the I_1 engine developed in these regimes, equal to 2.5 and 1.8 p.u., respectively).

Reference.

1. Yang, H., Shi, Y., Lau, J., & Zhang, T. (2014). An optimal control strategy based on virtual flow measuring for variable flow primary pumping. *HVAC&R Research. Winnipeg, Canada, 20*, 411-423.

2. Ankur, P. D., Rakesh, J. M., & Ajitsinh, R. C. (2014). Energy Conservation Using Variable Frequency Drive in Pumping Application. *RTEECE*, 121-126.

3. Radam, A. C., Vatavu, S., & Florea, C. (2017). Research regarding the operation of water pumping hydraulic systems. *Universitas Publiching Hause. Petrosanim Romania*, *2*, 25-32.

4. Wang, G., Zheng, X., & Kiamehr, K. (2017). Sequencing Control of Parallel Pumps in Variable-Flow Systems Using Wire-to-Water Efficiency. *ASHRAE Transactions*, *123*, 291-301.

5. Viholainen, J. (2014). Energy-efficient control strategies for variable speed driven parallel pumping systems based on pump operation point monitoring with frequency converters. *Lappeenranta,* Finland.

6. Os'kin, S. V., Didych, V. A., & Vozmilov, A. G. (2017). Key ways of energy saving in pump units for melioration and irrigation systems. *2017 ICIEAM IEEE. St. Petersburg, Russian Federation*, 212-216.

7. Bibik, O. V., Popovich, O. M., & Shevchuk, S. P. (2016). Energy-efficient regimes of the electromechanical system of a pumping unit for many superficial buildings. *Technichna electodinamika*, *5*, 38-45.

8. Volkov, V. A. (2018). Energy-saving control in start-braking regimes of a variable frequency asynchronous engine loaded with a centrifugal pump. *Electromehanichni I energozberigauchi sistemi,1*(41), 23-36.

9. Volkov, V. O. (2019). Optimization of times of start-braking regimes of frequency-regulated asynchronous engine with pumping load. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu, 1*(169), 90-98. DOI: 10.29202/nvngu/2019-1/11.

10. Leonov, B. S. (2013). *Variable Frequency Electric Pumping Units*.

Оптимізація загальних втрат енергії частотно-регульованого насосного агрегату при пуско-гальмівних режимах

В.О. Волков

Національний технічний університет „Дніпровська політехніка", м. Дніпро, Україна, e-mail: green_stone@ukr.net

Мета. Аналітичний розрахунок і оптимізація загальних втрат енергії для частотно-регульованого відцентрового насосного агрегату при пуско-гальмівних режимах.

Методика. Варіаційного числення, математичної інтерполяції й комп'ютерного моделювання.

Результати. Отримані аналітичні залежності, що дозволяють розрахувати та оптимізувати загальні втрати енергії частотно-регульованого відцентрового насосного агрегату в пуско-гальмівних режимах. Визначені квазіоптимальний вид тахограм і оптимальні значення тривалості часу розгону й гальмування цього агрегату, за яких забезпечується мінімізація його загальних втрат енергії в пуско-гальмівних режимах. Виконані приклади розрахунку цих втрат, а також гідравлічних, електромеханічних і енергетичних перехідних процесів для відцентрового насосного агрегату.

Наукова новизна. Уперше отримані аналітичні залежності для розрахунку в пуско-гальмівних режимах загальних втрат енергії частотно-регульованого насосного агрегату. Встановлено "U"-подібний вид залежностей загальних втрат енергії даного агрегату від тривалості часу його розгону й гальмування для різноманітних траєкторій зміни швидкості. Запропонована квазіоптимальна траєкторія зміни швидкості у вигляді часової функції гіперболічного синуса з варійованим значенням коефіцієнта в її аргументі, за якого загальні втрати насосного агрегату в пуско-гальмівних режимах мінімальні. Виконане порівняння загальних втрат енергії насосного агрегату для квазіоптимальної траєкторії швидкості з відомими іншими траєкторіями при різній тривалості часу пуско-гальмівних режимів, що дозволило оцінити досягнуте при цьому енергозбереження.

Практична значимість. Впровадження отриманих результатів дозволяє знизити до мінімально можливих значень непродуктивні втрати енергії для відцентрового насосного агрегату в пуско-гальмівних режимах.

Ключові слова: *частотно-регульований насосний агрегат, загальні втрати енергії, пуско-гальмівні режими*

Оптимизация общих потерь энергии частотно-регулируемого насосного агрегата при пуско-тормозных режимах

В.А. Волков

Национальный технический университет „Днепровская политехника", г. Днепр, Украина, e-mail: green_stone@ukr.net

Цель. Аналитический расчет и оптимизация общих потерь энергии для частотно-регулируемого центробежного насосного агрегата при пуско-тормозных режимах.

Методика. Вариационного исчисления, математической интерполяции и компьютерного моделирования.

Результаты. Получены аналитические зависимости, позволяющие рассчитать и оптимизировать общие потери энергии частотно-регулируемого центробежного насосного агрегата в пуско-тормозных режимах. Определены квазиоптимальный вид тахограмм и оптимальные значения длительности времени разгона и торможения этого агрегата, при котором обеспечивается минимизация его общих потерь энергии в пуско-тормозных режимах. Выполнены примеры расчета этих потерь, а также гидравлических, электромеханических и энергетических переходных процессов для центробежного насосного агрегата.

Научная новизна. Впервые получены аналитические зависимости для расчета в пуско-тормозных режимах общих потерь энергии частотно-регулируемого насосного агрегата. Установлен „U“-образный вид зависимостей общих потерь энергии данного агрегата от длительности времени его разгона и торможения для различных траекторий изменения скорости. Предложена квазиоптимальная траектория изменения скорости в виде временной функции гиперболического синуса с варьируемым значением коэффициента в ее аргументе, при которой общие потери насосного агрегата в пуско-тормозных режимах минимальны. Выполнено сравнение общих потерь энергии насосного агрегата для квазиоптимальной траектории скорости с известными другими траекториями при различной длительности времени пуско-тормозных режимов, что позволило оценить достигаемое при этом энергосбережение.

Практическая значимость. Внедрение полученных результатов позволяет снизить до минимально возможных значений непроизводительные потери энергии для центробежного насосного агрегата в пуско-тормозных режимах.

Ключевые слова: *частотно-регулируемый насосный агрегат, общие потери энергии, пуско-тормозные режимы*

Рекомендовано до публікації докт. техн. наук С. М. Тиховодом. Дата надходження рукопису 22.03.18.